School Outcomes and School Costs: A Technical Supplement*

Timothy J. Gronberg
Dennis W. Jansen
Lori L. Taylor
Kevin Booker

Texas A&M University
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An educational cost function uses statistical techniques to measure the systematic relationship between actual expenditures and educational outcomes given input prices and technological factors like student characteristics and school district size. As such, it is specifically designed to measure the kinds of cost differentials that characterize Texas. We believe it is the approach best suited for evaluating the cost of education in Texas. It is the approach taken in our initial report "School Outcomes and School Costs: The Cost Function Approach" (Gronberg et al. 2004). In this technical supplement, we describe the theoretical foundations and estimation specifics of that analysis, explore the sensitivity of the baseline model to alternative specifications, and extend the analysis to cover the 2003-2004 school year.

Cost Function Fundamentals

For a firm, the cost function models the relationship between firm costs, firm output, and input prices. The short run cost function may model the relationship between firm costs, firm output, the prices of variable inputs, and the level of fixed or quasi-fixed inputs.

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Reasons to consider using a cost function to estimate the cost of education in Texas:

- According to the fundamental principle of duality in production, the cost function summarizes all of the economically relevant information about the process of transforming inputs into outputs. That is, the estimated cost function allows a complete description of the technology available to a firm.
- The cost function allows estimation when output prices are unavailable or are not determined in a competitive market.
- The cost function more easily handles a multi-product firm than the alternative of directly estimating the technology via a production function.
- The cost function allows a relatively straightforward calculation of alternative cost indices for policy analysis.
- The cost function approach is applicable as long as firms are minimizing costs. It does not require profit maximization. (The ability to employ a cost function approach without assuming profit maximization is especially useful for nonprofit institutions such as public schools. Public schools may attempt to provide education services at minimum cost, but they are certainly not profit maximizing.)

Drawbacks to using a cost function:

- The cost function requires data on input prices. Such data are not always available.
- The cost function approach assumes that all firms are trying to minimize cost. As discussed below, statistical methods allow us accommodate school districts that are inefficient, but even this approach requires that the firms or institutions under study are

attempting to minimize costs of achieving a given level of output even though for some reason they may fail in the attempt. If school districts are not trying to minimize cost then cost function analysis can be misleading.

- The cost function approach is also subject to many if not all of the usual econometric concerns, from the possibility of simultaneous equations bias to concerns about mis-specification. We will discuss some of these concerns below.

The Texas Cost Function Model

School districts produce education outcomes—quantity and quality—using an education process that combines input factors that are purchased (for example, teachers and other personnel) with environmental input factors that are not purchased (for example, student skills acquired in an earlier grade). Thus, we model school district cost as a function of the outcomes produced, the prices of inputs, the characteristics of students and parents that directly influence the education production process, and other features that influence the education technology, such as school district size.

There exists a sizeable literature that finds that school districts do not all operate in an efficient, cost-minimizing fashion and that the degree of inefficiency varies considerably across districts. For example, Imazeki and Reschovsky find that "the average district in Texas is 59 percent as efficient as the most efficient districts in the state" (Imazeki and Rescovsky 2004a, 41). We use a stochastic frontier cost function model to accommodate this potential inefficiency. (See appendix A.)

Our model is a district-level cost function. The underlying conceptual assumption is that cost decisions are largely made at the district level, where district administrators allocate funds across campuses and across physical facilities and instructional programs. An underlying practical consideration is that state funds are allocated to school districts. A primary purpose of this study is to develop cost indices which could be used in the development of state funding formulas, so the district is the appropriate unit of observation.

Our analysis treats each school district as representing a separate observation of the educational process. In other words, each superintendent has potentially valuable insight into strategies for successfully allocating resources. Differences in the relationship between inputs and outputs that arise from differences in scale are captured by the specified and highly flexible relationship between expenditures and school district size.

We do not weight the estimation by school district size for a number of reasons. On theoretical grounds, it is inappropriate to weight some decision-making units more than others in a cost analysis. Cost analyses of hospitals, for example, do not weight by the number of patients (e.g. Bilodeau et al. 2000). Furthermore, on more practical grounds, pupil-weighting can introduce fatal statistical problems like those found in the Imazeki and Reschovsky analysis (Imazeki and Reschovsky 2004b). Pupil-weighting forces the model to reflect the cost patterns of the largest districts, introduces heteroskedasticity, has a tendency to introduce measurement bias, and is sensitive to outlier districts, all of which are statistical problems that can severely undermine the reliability of any econometric model (Taylor 2004).

We would note that the alternative of estimating campus level cost functions is severely hampered by a common cost allocation problem. School districts in Texas have differing accounting procedures for keeping track of central administration costs such that campus-level expenditures in some districts contain elements of central administration costs that are absent in campus-level expenditures in other districts. Some of this may occur because of the tremendous differences in school district sizes in Texas, with districts with less than 200 students and districts nearing 200,000 students. Central administration costs may well be more easily allocated to a central office when districts are larger. But even among districts of similar size, there are differences in where administrative personnel are located and hence differences in how such costs appear in district accounts.

Model Specification

The functional form for the Texas cost function model is a variant of the translog cost function. (It is a variant in that the model has been adapted to incorporate percentage changes without taking natural logs.) A primary advantage of the translog is its flexibility. Other popular functional forms—including the Cobb-Douglas specification used by Imazeki and Reschovsky (2004a, 2004b)—are restricted, special cases of the translog. Thus, if a restricted model is the best fit to the data, the translog estimation yields the same coefficients as would estimation of the restricted model. However, if there is a specification that fits the data better than the restricted model, the unrestricted, translog can find it while the restricted model cannot. The primary disadvantages of the translog specification are its complexity in evaluating marginal effects and the statistical concerns with multi-collinearity and over-parametrization due to the presence of many interaction terms involving the explanatory cost factors.

Our modified translog cost function has multiple output measures (number of students educated plus measures of output quality), one or more input prices, and a multitude of "environmental factors" and/or fixed factors of production. The left-hand-side variable is expenditures per pupil (E). The right-hand-side variables are the output measures (y_i), the input prices (w_j), and the environmental factors (x_k). All variables (except those expressed as percentages or percentage points) are in natural logs. Because the specification is a translog, all of the right-hand-side variables are interacted with each other and with themselves (meaning that all squared terms are included). Because school district size varies so greatly within Texas, other researchers have chosen to exclude the largest Texas districts from analysis (Imazeki and Reschovsky 2004a). Rather than take such an approach, we also include a cubic term for log enrollment. Including the cubic term adds to the flexibility of the model at virtually no cost; if the best fit to the data with respect to scale is quadratic (i.e., U-shaped) then the coefficient on the cubic term will be found to be zero.

The model for district expenditures per pupil with three output measures, two input prices, and eight environmental factors, is:

$$\ln\left(E_i\right) = a_0 + \sum_{i=1}^{3} a_i q_i + \sum_{i=1}^{2} b_i w_i + \sum_{i=1}^{8} c_i x_i + .5\sum_{i=1}^{3} \sum_{j=1}^{3} d_{ij} q_i q_j + \sum_{i=1}^{4} \sum_{j=1}^{2} e_{ij} q_i w_j$$

$$+ .5\sum_{i=1}^{2} \sum_{j=1}^{2} f_{ij} w_i w_j + \sum_{i=1}^{8} \sum_{j=1}^{2} g_{ij} x_i w_j + .5\sum_{i=1}^{3} \sum_{j=1}^{8} h_{ij} q_i x_j + .5\sum_{i=1}^{8} \sum_{j=1}^{8} k_{ij} x_i x_j + q x_1^3 + v_i + u_i$$

where $d_{ij} = d_{ji}$, $e_{ij} = e_{ji}$, $f_{ij} = f_{ji}$, $g_{ij} = g_{ji}$, $h_{ij} = h_{ji}$, and $k_{ij} = k_{ji}$. Because we explicitly allow for the possibility of school district inefficiency, the error term has two parts—a standard two-sided term to capture random un-modeled differences across districts (v_i) and a one-sided term to capture inefficiencies (u_i). (We note that if inefficiency is negligible (u_i=0) then the model collapses to a simple, ordinary least squares estimation.)

Calculating Marginal Effects

One drawback to using a translog specification is that it can be difficult to interpret the coefficient estimates. To aid in interpretation, most researchers calculate the change in cost arising from a small change in each of the explanatory variables. These implied marginal effects depend on the values of the explanatory variables and hence in general will differ for each district. It is common to report the marginal effects calculated at the mean of the values of the explanatory variables.

For example, the marginal effect of a change in the passing rate on expenditures per pupil, $me(q_1)$, is:

$$me(q_1) = a_1 + \sum_{j=1}^{3} d_{1j} q_j + \sum_{j=1}^{2} e_{1j} w_j + \sum_{i=1}^{8} h_{1i} x_i .$$

The marginal effect of a change in the wage on expenditure per pupil, me(w), and the marginal effect of a change in economically disadvantaged on expenditure per pupil, $me(x_1)$, are:

$$me(w_1) = b_1 + \sum_{i=1}^{3} e_{1i} q_i + \sum_{j=1}^{2} f_{1j} w_j + \sum_{i=1}^{8} g_{1i} x_i,$$

$$me(x_1) = c_1 + \sum_{j=1}^{2} g_{1j} w_j + \sum_{i=1}^{3} h_{i1}, q_i + \sum_{i=1}^{8} k_{1j} x_j.$$

The standard error of each calculated marginal effect is the square root of its variance. The standard errors on these calculated marginal effects depend on the variances and covariances of the estimated coefficients, as well as on the means of the explanatory variables. Appendix B presents the formula for calculating the variance of me(q_1), for example.

Estimation Specifics

We estimate our baseline cost model by pooling school district data for academic years 1998-99 through 2001-02. Due to concerns about cost function differences among districts with different service scope, we focus on districts that offer K–12 education. There are 974 of these in Texas, out of a total of 1,042 districts in Texas. (Of the others, 45 provide K–8 education, and 23 others provide K–6, K–5, or K–4 educations.) Six districts that have no tax base are excluded because they face a different incentive environment than other school districts. Of the remaining 968 K–12 districts, another 274 are missing one or more explanatory variables and must be excluded. Most of the districts that could not be included in the analysis because of missing data have too few students in each grade level to construct credible measures of student performance.

Table 1 summarizes the variables used in the estimation. Formal definitions of each variable are presented in appendix C.

Table 1. Variables Included in the Cost Function Analysis

Dependent variable
- Operating expenditures per pupil (log)

Outputs
- Change in average TAAS passing rate
- Percent completing an advanced course
- Percent above criterion on SAT/ACT

Input Prices
- Average monthly salary for beginning teachers (log)
- Average monthly salary for auxiliary workers (log)

Environmental Factors
- Prior TAAS passing rate
- District enrollment (log)
- Percent free lunch, non-high school
- Percent limited English proficient (LEP), non-high school
- Percent special education, less severe
- Percent special education, more severe
- Percent high school
- Miles to nearest major metro center (log)

All variables that are measured in dollars are converted to 1998 dollar equivalents to correct for inflation. The conversion to 1998 dollar equivalents was made using the Bureau of Labor Statistics' Employment Cost Index (ECI). We calculated the ECI for each school year as the average of the first two quarters of the calendar year and the last two quarters of the previous calendar year. Thus, the index for the 1998–99 school year is the average of the ECI for the last two quarters of 1998 and the first two quarters of 1999. Because labor costs have been rising faster than the costs of energy, technology, and other school supplies, the ECI will tend to overstate education sector inflation. However, education is a very labor-intensive industry, so the overstatement should be modest.

The cost function regression is a stochastic frontier regression with the one-sided error term specified as having a half-normal distribution. The regression is run on district observations for years 1999, 2000, 2001, and 2002. The regression sample is an unbalanced panel, with districts appearing in the years for which they have non-missing values for all the necessary variables. Because we use multi-year data, we also include year indicator variables. The regression uses Stata's "vhet" option to control for heteroskedasticity in the two-sided error term, with the variance function specified as a linear combination of two indicator variables, one for whether a district has less than 1,600 students, and the other for whether a district has less than 5,000 students. Small (less than 1,600 students) and mid-size (between 1,600 and 5,000 students) school districts receive special treatment under the Texas school finance formula.

The Baseline Estimation

Table 2 presents the coefficient estimates from our baseline estimation. From these coefficients, we calculate the marginal effect of each variable by evaluating the first derivative for that variable at the means of the other variables. Thus, the marginal effect for y_1 would be the coefficient on y_1, plus the coefficient on y_1y_2 times the mean of y_2, plus the coefficient on y_3 times the mean of y_3, and so on, plus 2 times the coefficient on y_1-squared times the mean of y_1. We note that because the estimation is not weighted, the appropriate means are the unweighted averages of the 2002 data for all of the districts included in the regression sample. Table 3 presents those unweighted means, while table 4 presents the estimated marginal effects

As table 4 illustrates, the baseline model is consistent with reasonable expectations about school district costs. Once we control for school district inefficiency, we find that costs increase as any of the outputs increase and that the cost of reaching any particular performance goal depends on educator wages, student needs, school district isolation, and school district size. (The relationship between expenditures and the change in the average TAAS passing rate is significant at the five percent level, as is the relationship between expenditures and the percent completing an advanced course. The relationship between expenditures and the percent scoring above criterion on SAT/ACT is significant at the 10 percent level.)

Costs increase as the share of students in the free lunch program increases and as the share of limited English Proficient (LEP) students increases. The estimation suggests that at the mean, a one percentage point increase in the share of free lunch students increases school district cost by 0.28 percent while a one percentage point decrease in the share of free lunch students decreases school district costs by 0.28 percent. Similarly, a one percentage point increase in the share of LEP students increases cost by 0.19 percent and a one percentage point decrease in the share of LEP students decreases costs by 0.19 percent.

We note that the coefficient on the cubic enrollment term is highly significant, indicating that it would be inappropriate to exclude this variable from the estimation. Including this factor, we find that costs fall with school district size. That is to say, we find no evidence of cost savings from breaking up large districts such as the Dallas or Houston ISDs. (Projected costs are higher in the Dallas and Houston ISDs than in many other districts, but the differential is attributable to student needs and geographic location rather than school district size per se.)

We do find evidence that there could be substantial cost savings from school district consolidation. The model suggests that the cost of achieving any given performance standard is four percent higher in a district with 200 students than it is in a district with 400 students, which in turn has costs four percent higher than in a district with 800 students. Interestingly, the model also suggests that the economies of scale are greater in a high wage environment than in a low wage environment, so the gains from consolidation in urban areas are greater than the gains from consolidation in rural areas.

Efficiency

The baseline model generates efficiency estimates for each district in our sample. Table 5 illustrates the distribution of estimated efficiency values. The distribution is heavily skewed, with three quarters of the observations indicating an efficiency of 92 percent or better. The minimum efficiency value is 74 percent. The average level of efficiency is 93 percent.

These relative efficiency measures need to be interpreted with caution, as does the separation of spending differences into those due to cost factor differences and those due to relative efficiency.

The measurement of efficiency in producing a set of outcomes is directly linked to the particular set of performance measures that are included in the cost model and the particular set of input measures. For example, if only a math and reading performance measure is included in the estimated cost function model but a district devotes significant resources to music and athletic programs (and if those programs have little correlation with math and reading) then this district will be identified as inefficient, even if all of its programs are being produced as efficiently as possible.

Unobserved variation in important unmeasured quasi-fixed factors could also muddy the interpretation of the efficiency estimates. In particular, if capital varies widely across districts, and if increased capital leads to increased educational achievement, then two otherwise identical districts that vary in the size of their capital stock will show one district realizing higher levels of educational achievement.

Finally, we would point out that the best-practice frontier estimated here is based upon observed district decisions made within the incentive environment in place at the time of observation. Changes in the institutional environment, with associated changes in fundamental incentive structures, would be expected to shift the frontier, both actual and estimated. Changes in the incentive environment would also be expected to impact how far the districts are located from the frontier.

Calculating Predicted Costs

To calculate predicted costs for each district, we first calculate the predicted cost for a benchmark district, i.e., a district with the mean value for each right-hand-side variable. We then use the marginal effects to calculate the costs of performance in each district according to its deviation from the benchmark district.

Relying on the marginal effects to generate district-level cost estimates rather than each specific coefficient estimate sacrifices some precision in the district-level cost estimates, but arguably enhances the usefulness of the cost function analysis in the design of a school finance formula. In the full blown cost function, the cost impact of having an additional student with limited English proficiency depends on all of the right-hand-side characteristics of the district, differs from one district to another, and changes from one year to the next as those other characteristics change. A finance formula based on such a model would be a black box wherein each district would have its own unique formula weight and only researchers would understand the underpinnings of that weight. Given the lack of transparency and the potential for mischief inherent in district specific weights, we don't advocate using this approach to a finance formula for Texas. In contrast, using the estimated marginal effects to generate cost estimates is equivalent to using a system of standard cost adjustments that apply to all districts. This system is more transparent, because predicted costs for each district are easily generated based on the differences between the district's characteristics and those of the benchmark district. That said, it is important to note that it could be highly problematic to take any of these marginal effects out of context of the comprehensive model or adopt them piecemeal. The cost estimates we generated are based on a coherent set of marginal effects treated as cost adjustments; a distribution formula that used certain adjustments without others would most likely fail to achieve an appropriate distribution

Cost in a Benchmark District

The benchmark district has as its characteristics the mean from the regression sample for the 2002 academic year. In other words, table 3 describes the characteristics of the benchmark district. Each of the non-interacted variables is set to the district mean, then the variables are interacted and squared. The benchmark district's predicted cost is calculated by multiplying each of these right-hand-side variables by the coefficient on that variable from the cost function regression and adding up each of the terms on the right hand side of the cost function. Finally, we presume that the benchmark district has average inefficiency. Thus, we add the mean one-sided error for 2002 to the benchmark (log) predicted cost.

Predicting Costs per District

In order to generate cost projections based on the marginal effects, we linearize the cost function before generating cost predictions for each district. The predicted cost for each district is the predicted cost for the benchmark district (calculated as described above) plus each of the linearized coefficients multiplied by the difference in value between the specific district and the benchmark district. However, to ensure that our cost estimates are based on factors outside of school district control, we replace the average actual wages with the corresponding predicted wages.

The linearization is a first order Taylor expansion in all variables except for district enrollment and auxiliary worker salary. For enrollment, we also include the second and third order terms, and for auxiliary worker salary we include the interaction term of auxiliary worker salary and district enrollment. These two adjustments were made to reduce the loss in accuracy associated with the simplifying linearization assumption. The relationship between cost and enrollment is clearly nonlinear, and including the enrollment-auxiliary worker interaction greatly improves the match between the predicted cost estimates from the (quasi-) linearized model and those of the full translog model for the largest districts. Figures 1 and 2 display the distribution of predicted costs under the full translog model, the linearized model excluding the auxiliary-enrollment interaction adjustment, and the linearized model including the auxiliary-enrollment interaction adjustment. As the figures illustrate, excluding the interaction term dramatically degrades the match between the full model predicted costs and the linearized model predicted costs for the two largest districts, Dallas and Houston.

Checking for Endogeneity

Arguably, the wages that school districts are willing to pay, the outcomes they intend to achieve, and the total they are willing to spend are determined simultaneously. If so, then one must consider the possibility that wages and outcomes are endogenous. Instrumental variables analysis is the customary diagnostic tool for endogeneity problems.

The choice of instruments for outputs is not obvious. A good instrument for each output must not only be correlated with the output, but uncorrelated with the error term from the cost function equation. For example, property wealth (which was used as an instrument for school outputs by Imazeki and Reschovsky 2004b) would be an inappropriate instrument for output because it is likely correlated with school district facilities and therefore with the cost function error term. Furthermore, when the variables to be instrumented vary over time in a panel, the instruments must also vary over time.

The determinants of consumer demand for education are commonly used as instruments for educational output. Unfortunately, voter characteristics such as the percentage of residents with college degrees or school age children, are not measured frequently enough to be good instruments for panel analysis. Measures of a district's ability to pay for schooling change over time, but are not good instruments because they are undoubtedly correlated with the cost function errors. To capture the keeping-up-with-the-Jones's aspect of educational demand, we used the enrollment-weighted average value of each output in the surrounding districts as an instrument for the district's actual output.

Appropriate instruments for wages are much easier to identify. For each wage series (teacher wages and auxiliary wages) we used the predicted wage as an instrument for the actual wage.

Because instrumental variables analysis is not commonly done with a stochastic frontier specification, we conducted our analysis of endogeneity using two stage least squares to estimate the translog function.

First we tested for endogeneity in the outputs and their interactions. The potentially endogenous variables were the three output variables, their squared terms, and their interactions with each other and with the other variables in the regression. The instruments are the three output instruments, their squared terms, and their interactions with each other and with the exogenous variables in the regression. The Hausman test cannot reject the hypothesis that instruments unnecessary. (The probability value of the test statistic is above 0.9999.)

We also tested for endogeneity in the wages, and in both the outputs and the wages together, each with a similar specification, instrumenting for the endogenous variables and their squares and interactions with the instruments, their squares and their interactions. In each case the Hausman test failed to reject the hypothesis that instruments are unnecessary. (The probability value of the test statistic is above 0.9545 for the former, and above 0.9999 for the latter.)

Alternative Specifications

In addition to our baseline model, we also estimated a number of alternative models designed to shed light on the primary specification. Table 6 compares the marginal effects from these alternative specifications with the baseline estimation. Table 7 compares the predicted expenditures under the various scenarios. For each alternative specification, we use the linearized model to predict the cost of achieving average performance on each of the outcome measures in each of the districts, assuming an average level of school district efficiency. The descriptive statistics in table 7 are pupil-weighted so that the percentage difference between means across models is also the percentage difference for the state as a whole.

Model B

The first alternative model, Model B, is the one reported in the March 2004 report (Gronberg, Jansen, Taylor and Booker 2004). In the earlier report, we inadvertently excluded expenditures in object codes 6401-6499 (other operating costs) from the measure of expenditures per pupil. As table 6 illustrates, the omission had little impact on the coefficient estimates. The correlation between cost projections based on Models A and B is .995. However, as expected, broadening the definition of expenditures increases the predicted cost of achieving average performance.

Model C

The second alternative model explores the sensitivity of the model to the designated measure of student poverty. The baseline model uses the percent of non-high school students receiving free lunches as the measure of poverty. The focus on non-high school students is intuitively appealing because anecdotal evidence suggests that high school students are less likely to identify themselves as needy, even when they are. However, questions might be raised about the focus on free lunch rather than free and reduced lunches. Model C uses the percent of non-high school students receiving either free or reduced lunches as the measure of student poverty. As table 6 indicates, when the definition of student poverty is broader, the marginal impact of an increase in student poverty on cost is smaller (the marginal effect of a percentage point change in low income share falls from 0.28 to 0.24). The estimated cost of achieving average performance in Texas drops by a negligible 0.2 percent (on average) when student poverty is defined using free and reduced lunch participation rather than free lunch participation (table 7).

Model D

The third alternative specification explores the impact of using predicted wages rather than actual wages to estimate the cost function. In our judgment, it is more appropriate to estimate the cost function using a measure of actual rather than predicted wages. However, other researchers favor using predicted wages as the measure of labor cost for both the estimation of the model and the generation of predicted costs (e.g., Imazeki and Reschovsky 2004b). As Model D illustrates, using predicted rather than actual wages yields a model that does not fit the data nearly as well. (The log likelihood value falls from 2,755 to 2,568.) Using predicted wages in the estimation tends to increase the estimated marginal effect of wages and widen the range of cost projections. The highest cost estimate is higher and the lowest cost estimate is lower when predicted wages are used in the estimation. On average, using predicted wages in the estimation generates somewhat lower cost estimates than the baseline model. Using predicted wages in the estimation yields substantially lower cost estimates for large school districts.

Model E

Because of its flexibility, the translog cost function can nest within it more restrictive specifications. One such specification is the Cobb-Douglas specification. The pure Cobb-Douglas specification restricts the coefficients to zero on all of the squared terms and all of the interaction terms in the translog specification. As our fourth alternative specification, Model E estimates a modified Cobb-Douglas model, where school district size continues to have squared and cubic terms, but all other squared or interaction terms are set equal to zero. As table 6 illustrates, this Cobb-Douglas model is clearly inferior to the translog. The log likelihood is much lower for the Cobb-Douglas specification, and a log-likelihood test easily rejects the Cobb-Douglas restrictions. In addition, some of the coefficients are implausible. For example, the Cobb-Douglas model indicates that the marginal cost of additional high-need special education students is negative, and that the marginal cost of an improvement in TAAS passing rates is essentially zero.

One criticism of the translog approach is that such an extremely flexible functional form may be "overparameterized". In particular, it has been suggested that the large number of interaction terms obscures a much larger cost impact of student need. Indeed, our estimates of the marginal effect of student need are lower than those found in other states. However, if overparameterization were causing such a problem, the marginal effects from the more-parsimonious Cobb-Douglas specification would be larger than those in the baseline model. Instead, the Cobb-Douglas specification yields lower estimates of the marginal effects of student poverty and limited English proficiency than does the baseline model. Therefore, the relatively low marginal effects of poverty and LEP are unlikely to arise from over-parameterization. Given that such a large fraction of Texas students are LEP (15 percent) or low income (50 percent) the comparatively low marginal effects at the mean may simply reflect the large extent to which the costs of student need are already built into the baseline estimates.

Model F

The fifth alternative explores the impact of the frontier estimation strategy. Model F estimates the baseline translog, but without the assumption of frontier errors. As table 6 illustrates, the frontier estimation approach (Model A) fits the data better than does an ordinary least squares estimation (Model F). Not surprisingly, Model A yields systematically smaller cost estimates than does Model F. However, because we presume average efficiency in the cost projections rather than some higher level of efficiency, the difference is generally modest.

Model G

One point of criticism of the baseline model is the way we measure performance on the TAAS. For each grade level, we calculate the average percent passing as one half of the sum of the percent passing reading plus the percent passing mathematics. Thus, the percent passing math and reading in the fifth grade is the percent of fifth-graders passing math plus the percent of fifth-graders passing reading, divided by two. Alternatively, one could measure the percent passing reading and math as the percent passing both reading and math, and the change in percent passing as the change in percent passing both reading and math. Model G uses this alternative calculation strategy to measure both the change in average TAAS passing rate and the prior TAAS passing rate. This alternative measure of passing rates yields a model that indicates a substantially lower marginal effect of increasing the passing rate. On the other hand, because the average annual gain in the percent passing both tests is greater than the average annual gain in the average passing rate, the cost of achieving average value added is roughly equivalent under either specification.

Model H

Other critics have argued that the Texas accountability system requires that all student groups pass the standardized tests, and that therefore, the primary interest should be in the cost of getting the most disadvantaged students to pass the test, not just the average student. This dimension of the Texas accountability system is too complex to be fully captured in any empirical model. One strategy for shedding additional light on the distribution of student performance within school districts is to estimate costs on the basis of the performance of student subgroups. Therefore, Model H uses as its measure of value added the change in average passing rates for economically disadvantaged students rather than the change in average passing rates for all students. Economically disadvantaged students have passing rates that generally trail those of other student groups and most school districts have enough economically disadvantaged students to be able to calculate reliable estimates of the change in passing rate (All other student subgroups, such as students with limited English proficiency, are much less pervasive and we would lose too many districts from the estimation to generate a reliable model if we only included districts with data for those subgroups. As it is, we lose 144 school district when estimating Model H because their economically disadvantaged population is too small.) As table 6 illustrates, replacing the average passing rate change with the passing rate change for low-income students dramatically lowers the estimated marginal effect. The marginal effect on expenditures of a change in the average passing rate for economically disadvantaged students is not statistically significant. While the overall improvement in passing rate is an imperfect measure of school performance, for the purposes of cost analysis it appears to be a better measure of school outcomes than is the improvement in passing rate for low-income students.

Model I

Model I explores the impact of school district size on the cost function. Texas has a disproportionate number of small school districts, and the educational process in small districts might differ enough from the educational process in larger districts that they should not be examined in the same model. Model I restricts the sample by excluding districts with enrollments below 1,600. As table 6 shows, focusing on midsize and larger districts lowers somewhat the estimated marginal effect of a change in the passing rate. The smaller sample required to make such calculations also leads to a substantially higher standard error for the estimated marginal effect. Again, one cannot conclude that the marginal effect is different from zero when the sample is so restricted. Interestingly, excluding the smaller districts from the sample generates lower cost estimates for both the largest and the smallest school districts. The cost estimates for Dallas and Houston ISDs, for example, are more than five percent lower in Model I than in the baseline model.

Model J

As the above models illustrate, the baseline specification is quite robust. It fits the data better than a number of alternative specifications and is generally indifferent to small changes in the way that variables are measured. In particular, there is little to be gained analytically from using free lunch participation as the indicator of student poverty rather than free and reduced lunch participation, or from using the average passing rate change rather than the average change in the percent of students passing both math and reading. (The correlation between predicted expenditures from models A and C is .9971 while the correlation between predicted expenditures from models A and G is .9998.)

Model J presents a "refined" version of the baseline model that combines models C and G. Student poverty is measured as the share of non-high school students receiving either free or reduced price lunches, and student performance is measured as the average change in the percentage of students passing both math and reading. As table 7 illustrates, these changes substantially lower the estimated marginal effects of increasing the passing rate and of increasing the share of eligible students scoring above criterion on the SAT or ACT. The SAT/ACT measure is no longer statistically significant, and the TAAS passing rate measure is significant only at the 10 percent level. However, the cost projections are virtually identical. The correlation in cost projections between Models A and J is .9967, and the difference in the total projected cost of average performance implied by the two models is only 0.1 percent (see table 7).

Extending the Analysis to 2003-04

All of the above models draw on data from the 1999–2002 school years. Preliminary data for the 2004 school year are now available. Therefore the last three alternative specifications—Models K, L, and M–draw on data from the 2003–04 school year.

Data availability issues force the 2003–04 analysis to differ from the 1999–2002 analysis in three key respects. First, the measure of school district expenditures is different. Actual financial data for 2004 are not yet available. Therefore, the 2004 analysis is based on budgeted expenditures. In prior years, budgeted expenditures have differed from actual expenditures in systematic ways. Budgeted expenditures were lower than actual expenditures in most districts, and the gap between budgeted and actual expenditures was bigger for small districts than for large ones. To correct for systematic differences between actual and budgeted expenditures, we adjust each district's budgeted expenditures for 2004 by the ratio of actual to budgeted expenditures in 2003. Thus, if the budgeted per pupil spending in 2004 is 2.5 percent above the budgeted 2003 level, then we presume that actual per-pupil spending in 2004 will also be 2.5 percent above the actual 2003 level. This adjustment yields a noisy estimate of actual expenditures, albeit one that should be closer to the truth than the unadjusted budget figures. However, the imprecision with which expenditures must be measured will likely increase the imprecision in the estimation.

Second, the outputs are different. In 2003, Texas replaced the TAAS with the Texas Assessment of Knowledge and Skills (TAKS). The TAKS is widely recognized as a more challenging test than the TAAS. However, there are only two years of data on the new test. Therefore, the 2004 models use only a one-year change in passing rates rather than a three-year moving average change as in the other specifications. The moving-average approach reduces the statistical noise in the outcomes measures, so any passing rate change is less precisely measured in the 2004 models than it was in the corresponding 1999–2004 model. In addition, the 2003 TAKS was administered in a pilot program without consequences to students or districts for their performance on that test. Therefore, it may represent a particularly noisy baseline from which to measure performance gains. On the other hand, the TAKS passing rate change includes the newly available data on ninth-grade and eleventh-grade performance, so it covers changes in passing rates for all grades five to 11 and permits us to measure annual performance gains.

Interestingly, the average change in TAKS passing rates is substantially greater than the average change in TAAS passing rates, despite the fact that the TAKS data reflects the change in passing rates from one year to the next while the TAAS data reflects changes in passing rates over a two-year period (such as from the eighth–grade passing rate in 2000 to the tenth–grade passing rate in 2002). On average, districts posted a 4.2 percentage point increase the in the average share of students passing both math and reading on the TAAS and a 6.3 percentage point increase in the average share of students passing both math and reading on the TAKS.

The switch to TAKS is not the only relevant output change. Because 2004 data on advanced courses and SAT/ACT scores are not available, we substitute data on advanced courses and SAT/ACT scores from 2003. For consistency, we do not use a moving average for any of the outcomes measures in the 2004 models.

In addition, Model M illustrates the impact of including the school district's completion rate as an outcome measure. The Texas accountability system makes student completion an important school district objective, so there undoubtedly is interest in the relationship between completion rates and school district expenditures. Analyses over the 1999–2002 time period found a perverse and implausible relationship between the dropout rate and expenditures, but the measure of student completion rates used in the accountability system has changed. Therefore, Model M explores anew the relationship between completion rates and school district expenditures.

Finally, the change in testing environment forces us to rely on a single year of data to estimate the 2004 models rather than the four years of data used to derive the baseline models. Where the baseline models reflected only cost patterns that persisted for four years, the 2004 models will reflect both persistent and transitory patterns. Therefore, while the 2004 models are preferable if one believes that the cost patterns have shifted with the testing environment, they are inferior to the baseline model if one believes that cost patterns are relatively stable.

Model K

Table 6 presents the marginal effects from the three models based on 2004 data. The first model, Model K, directly replicates Model J using 2004 data. As the table illustrates, Model K is generally consistent with the refined baseline model of school district costs. Costs are higher where wages are higher and where student need is higher. Remoteness drives up cost, but size drives cost down. The marginal effects of student poverty and limited English proficiency in Model K are within two standard errors of their counterparts in Model J. The marginal effect of an increase in percent high school students is much smaller for 2004, a pattern which may reflect the introduction of an eleventh grade TAKS test or may reflect a bias in the way we impute actual expenditures from budgeted expenditures. (It does not appear to reflect our reliance on lagged scores for the two high school performance indicators. When we exclude the advanced courses and SAT/ACT indicators from the 1999–2002 model, the marginal effect of percent high school remains above .65. The change also does not reflect differences in the sample of districts across the two time periods. When we restrict the analysis to only those districts for which we have complete data for both the 1999–2002 period and the 2003–04 period, we find the same difference in marginal effects across models.) The predicted expenditures from the two models are highly correlated, although the predicted cost of average performance is somewhat higher for Model K (see table 7).

The most important divergence between the two models is in the marginal effect of a change in the outcome measures. The marginal effect on cost of increasing passing rates is half as large in Model K as the effect in Model J, and not statistically significant. The marginal effect of scoring high on the SAT/ACT switches sign and also becomes statistically insignificant. It is probable that much of the change in estimated marginal effects reflects the larger magnitude of the average annual gain and the much lower precision in the estimation of passing rate changes. However, it may also indicate that passing rate gains are cheaper in the TAKS environment because we are in the initial years of a new testing regime; in the TAAS world, passing rates regularly exceeded 90 percent and it was very difficult to raise passing rates.

Model L

Model L is a bare-bones version of Model K. It includes only one outcome measure: the change in TAKS passing rates. Even in this stripped down version, the marginal effect of a change in TAKS passing rates is not statistically significant, although the estimated marginal effect increases. Given the imprecision inherent in the 2004 models, all of the marginal effects are within two standard errors of their estimates from the refined baseline model.

Model M

Model M illustrates the impact of introducing the completion rate for all students as a measure of school district performance. The completion rate is the number of graduates, GED recipients, and continuers expressed as a percentage of total students in their class. Among district in the estimation sample, the average completion rate is 96.8 percent, and most districts have completion rates above 90 percent. As table 6 illustrates, the coefficient on the completion rate is perverse, indicating that an increase in the completion rate is associated with a fall in school district cost. Given the implausibility of such a model, we reject it.

Evaluating the Cost of Meeting Performance Standards

One of the attractions of the cost function approach is that it can be used to generate estimates of the cost associated with meeting any reasonable performance standard. In the above discussion, that standard was the average performance of Texas school districts—18 percent completing advanced courses; 13 percent of the eligible students scoring above criterion on the SAT or ACT; and either a three percentage point increase in the TAAS average passing rate, a four percentage point increase in the average share passing both math and reading on the TAAS, or a six percentage point increase in the average share passing both math and reading on the TAKS.

The cost function analysis can also be used to predict the cost of achieving a particular passing rate. In our March 2004 report, we estimated the cost of achieving a TAKS-equivalent average passing rate of 55 percent, together with the state averages for the percent taking advanced courses and scoring well on the SAT/ACT. Here we compare the cost estimates from the baseline model with those generated by alternative specifications.

The concept of a TAKS-equivalent passing rate requires some explanation. The baseline models relate school district expenditures to student performance gains on TAAS, not TAKS. We translated TAAS performance in 2002 into TAKS-equivalent performance following a Texas Education Agency (TEA) conversion schedule. For example, the schedule indicates that where a score of 70 would have been high enough to pass the TAAS, a third grader would need to have scored an 82 to pass the math test at the TAKS standard that will be in place for 2005. (The 2005 grading standard is the panel recommendation.) We re-graded each individual student's TAAS test using the grading standards given in the conversion table, and, using these re-graded exams, calculated the TAKS-equivalent average passing rate for each district for 2002. (All students in the district with test score information were included in this calculation, not just those for whom a prior test score could be determined.) District TAKS-equivalent average passing rates at the panel recommendations standard ranged from 19 percent to 88 percent, with an average of 53 percent. Only six districts had TAKS-equivalent average passing rates above 80 percent.

At the time of our analysis, the TEA had not yet released its accountability standards for 2004. However, it was reasonable to expect that the passing rate required for a school district to be considered academically acceptable would be no higher in the TAKS world than it had been in the TAAS world. In the final year of TAAS testing, 55 percent of each type of student had to pass TAAS for a district to be considered acceptable. (In earlier years, the standard had been as low as 25 percent.) The published standards for 2004 indicate that for a district to be academically acceptable, 50 percent of each type of student must pass the TAKS reading/language arts exam, and 35 percent of each type of student must pass the math exam.

In addition, Texas's Consolidated State Application Accountability Workbook details the state's plans for compliance with the No Child Left Behind Act of 2001. The workbook sets out a progression of increasing passing rates on TAKS. The designated goal for 2006 is that the percent of students in all grades and demographic categories passing the TAKS be 53.5 percent for reading/language arts and 41.7 percent for math.

Given these figures, we chose to evaluate the cost of having 55 percent of students passing TAAS at the TAKS level. We calculated the gap between each district's TAKS-equivalent average passing rate and 55 percent. We then calculated the cost of producing at least the state average on the three outcome measures, plus whatever additional value added is required to close this performance gap. Thus, if a district had a TAKS-equivalent average passing rate of 52 percent (or higher), then we calculated the cost of producing a three percentage point increase in TAAS average passing rates (together with the state averages for advanced courses and SAT/ACT performance). If the district had a TAKS-equivalent average passing rate below 52 percent, then we calculated the cost of achieving enough percentage point increases in the passing rate to bring the district up to a 55 percent average passing rate. For example, if the district's TAKS-equivalent average passing rate was 50 percent, then we calculated the cost of achieving a five percentage point increase in the passing rate. In so doing, we implicitly assumed that the cost of achieving a one percentage point increase in TAKS passing rates was no higher than the cost of achieving a one percentage point increase in TAAS average passing rates. Such an assumption was consistent with the basic economic concept of diminishing returns and has been borne out by the marginal effects estimates from the 2004 models.

The baseline model indicated that the average cost of achieving at least a TAKS-equivalent average passing rate of 55 percent in 2002 (together with the state averages for percent taking advanced courses and scoring well on the SAT/ACT) was $6,389 per pupil on average (in 2004 dollars, weighted by 2004 enrollment). On average, per-pupil spending in 2002 was $6,561 (in 2004 dollars weighted by 2004 enrollment).

For comparison, we estimated the cost of achieving at least an average passing rate of 55 percent using the refined baseline model and the preferred models from the 2004 data (Models J, K, and L). As with the baseline model, we calculate costs assuming that each district produces the average value added, plus whatever additional value added is needed to bring the designated passing rate up to 55 percent.

Model J, the refined baseline estimates, relies on the same assumptions embedded in the baseline analysis, but differs in that districts are expected to close a larger gap than in the baseline analysis. Where the TAKS-equivalent average passing rate was 53 percent in 2002, the TAKS-equivalent average share of students passing both math and reading was 42 percent. Model J indicates that achieving at least an average of 55 percent passing both math and reading would cost $6,410 per pupil, on average.

Because Models K and L are based on actual TAKS performance, there is no need to calculate a conversion factor or make any additional assumptions about the marginal effect of a percentage point increase in the passing rate. Model K, the full replication of the refined baseline model, indicates that the cost of achieving at least an average of 55 percent passing both math and reading (and the state average on the other two outcomes measures) was $6,483 on average (see table 8). Model L indicates that the average cost of achieving that performance standard was $6,395. Our working estimate of actual per-pupil spending in 2004 averaged $6,513, a level that is roughly comparable after inflation adjustment to the average expenditure level for 2002, and a slight decline from the average real expenditure level of $6,725 in 2003.

The cost estimates from the refined baseline model are remarkably similar to those from the 2004 models. Figures 3a and 3b plot each set of 2004 cost estimates against the refined baseline. As the figures reveal, the cost estimates are most divergent for districts with higher cost estimates, which generally mean smaller districts. We note that district characteristics in 2002 are used to estimate the baseline models, while district characteristics in 2004 are used to estimate the alternative models. Because the characteristics of small districts are more volatile than those of larger districts, a noticeable difference in cost estimates for the smallest districts would be expected even if the models were identical.

To explore this point further, figures 4a and 4b plot the cost estimates against school district size. Here, we can clearly see that the cost estimates are most divergent for small districts. Model L, the single-output model, in particular generates noticeably higher estimates for small districts than does Model J, the refined baseline model.

In addition to the analysis at a 55-percent passing rate, there is considerable interest in the cost associated with higher performance levels. The baseline model was not used calculate the costs associated with higher performance levels because making such projections would have required us to extrapolate well beyond the available data.

There is no such problem with the 2004 models. While the TAKS-equivalent average share of students passing both math and reading in 2002 was 42 percent, the average share of students passing both math and reading in 2004 was 60 percent (both at the panel recommendations). There were more than 150 districts where the average passing rate in reading and math exceeded 70 percent in 2004; there were only 13 districts with a TAKS-equivalent passing rate in both math and reading above 70 percent in 2002.

Table 8 presents cost estimates at the 70-percent and 80-percent TAKS passing rates. As the table illustrates, the models suggest that the additional cost required to raise average passing rates to the 70-percent or even 80-percent level is relatively modest. In no case is the average cost estimate more than 1 percent above the average expenditure level of $6,513. The relatively small marginal cost associated with raising performance on TAKS is a rather surprising byproduct of the very low marginal effects estimates for the TAKS test and the relatively large number of districts already performing close to the 70-percent passing rate standard. However, given the imprecision with which the 2004 models are estimated, the actual cost associated with such performance levels could be substantially higher or lower than those presented here. We recommend re-estimating models K, L and M once the actual financial data for 2004 become available.

Table 2. The Cost Function Coefficients			
Variable	**Coefficient**	**Std. Error**	**P > [z]**
Intercept	0.7458	13.2752	0.9550
Change in Average TAAS Passing Rate	8.3970	12.8073	0.5120
% Completing an Advanced Course	-12.6898	3.7375	0.0010
% Above Criterion on SAT/ACT	-0.7817	4.2629	0.8550
Prior TAAS Passing Rate	3.7364	5.7978	0.5190
Average Monthly Salary for Beginning Teachers (log)	6.9810	3.1364	0.0260
Average Monthly Salary for Auxiliary Workers (log)	-6.2628	2.0905	0.0030
District Enrollment (log)	-0.1303	0.2881	0.6510
% Free Lunch (non-high school)	5.8509	1.8568	0.0020
% Limited English Proficient (non-high school)	-7.8486	3.0108	0.0090
% Less Severe Special Education	8.1103	8.0456	0.3130
% High school	-1.5030	8.5437	0.8600
Miles to Nearest Metro Center (log)	-0.8730	0.3435	0.0110
% Severe Disability Special Education	32.0760	19.2050	0.0950
Change in Average TAAS Passing Rate X % Completing an Advanced Course	3.1094	1.8061	0.0850
Change in Average TAAS Passing Rate X % Above Criterion on SAT/ACT	2.5364	2.5038	0.3110
Change in Average TAAS Passing Rate X Prior TAAS Passing Rate	0.3749	2.9194	0.8980
Change in Average TAAS Passing Rate X Average Monthly Salary for Beginning Teachers (log)	-1.4966	1.6153	0.3540
Change in Average TAAS Passing Rate X Average Monthly Salary for Auxiliary Workers (log)	0.3870	1.0366	0.7090
Change in Average TAAS Passing Rate X District Enrollment (log)	-0.0213	0.1414	0.8800
Change in Average TAAS Passing Rate X % Free Lunch (non-high school)	-1.3857	0.7906	0.0800
Change in Average TAAS Passing Rate X % Limited English Proficient (non-high school)	0.5629	1.0815	0.6030
Change in Average TAAS Passing Rate X % Less Severe Special Education	-7.5886	3.7517	0.0430
Change in Average TAAS Passing Rate X % High school	4.8681	4.5847	0.2880
Change in Average TAAS Passing Rate X Miles to Nearest Major Metro Center (log)	-0.2119	0.1585	0.1810
Change in Average TAAS Passing Rate X % Severe Disability Special Education	19.9720	9.4727	0.0350

Table 2. The Cost Function Coefficients			
Variable	**Coefficient**	**Std. Error**	**P > [z]**
% Completing an Advanced Course X % Eligible Students Above Criterion on SAT/ACT	-0.4119	0.6253	0.5100
% Completing an Advanced Course X Prior TAAS Passing Rate	1.8375	0.8296	0.0270
% Completing an Advanced Course X Average Monthly Salary for Beginning Teachers (log)	2.0020	0.5036	0.0000
% Completing an Advanced Course X Average Monthly Salary for Auxiliary Workers (log)	-0.7442	0.3457	0.0310
% Completing an Advanced Course X District Enrollment (log)	0.0416	0.0404	0.3030
% Completing an Advanced Course X % Free Lunch (non-high school)	-0.2773	0.2286	0.2250
% Completing an Advanced Course X % Limited English Proficient (non-high school)	-0.5371	0.3909	0.1690
% Completing an Advanced Course X % Less Severe Special Education	0.7107	1.1472	0.5360
% Completing an Advanced Course X % High school	-0.2234	1.2836	0.8620
% Completing an Advanced Course X Miles to Nearest Major Metro Center (log)	0.1947	0.0410	0.0000
% Completing an Advanced Course X % Severe Disability Special Education	0.9080	3.0847	0.7680
% Above Criterion on SAT/ACT X Prior TAAS Passing Rate	0.2947	1.1115	0.7910
% Above Criterion on SAT/ACT X Average Monthly Salary for Beginning Teachers (log)	-0.2361	0.5028	0.6390
% Above Criterion on SAT/ACT X Average Monthly Salary for Auxiliary Workers (log)	0.3970	0.3662	0.2780
% Above Criterion on SAT/ACT X District Enrollment (log)	-0.0665	0.0426	0.1180
% Above Criterion on SAT/ACT X % Free Lunch (non-high school)	1.3504	0.3266	0.0000
% Above Criterion on SAT/ACT X % Limited English Proficient (non-high school)	-1.9238	0.5626	0.0010
% Above Criterion on SAT/ACT X % Less Severe Special Education	4.3875	1.5362	0.0040
% Above Criterion on SAT/ACT X % High school	-0.0270	1.5368	0.9860
% Above Criterion on SAT/ACT X Miles to Nearest Major Metro Center (log)	-0.2069	0.0503	0.0000

Table 2. The Cost Function Coefficients			
Variable	**Coefficient**	**Std. Error**	**P > [z]**
% Above Criterion on SAT/ACT X % Severe Disability Special Education	-5.4581	2.9839	0.0670
Prior TAAS Passing Rate X Average Monthly Salary for Beginning Teachers (log)	-1.1492	0.7454	0.1230
Prior TAAS Passing Rate X Average Monthly Salary for Auxiliary Workers (log)	0.4984	0.4747	0.2940
Prior TAAS Passing Rate X District Enrollment (log)	-0.0169	0.0637	0.7910
Prior TAAS Passing Rate X % Free Lunch (non-high school)	-0.6050	0.3805	0.1120
Prior TAAS Passing Rate X % Limited English Proficient (non-high school)	0.3743	0.4918	0.4470
Prior TAAS Passing Rate X % Less Severe Special Education	-4.9285	1.7616	0.0050
Prior TAAS Passing Rate X % High school	5.9094	2.2009	0.0070
Prior TAAS Passing Rate X Miles to Nearest Major Metro Center (log)	-0.0978	0.0708	0.1670
Prior TAAS Passing Rate X % Severe Disability Special Education	7.7267	3.9119	0.0480
Average Monthly Salary for Beginning Teachers (log) X Average Monthly Salary for Auxiliary Workers (log)	0.7066	0.2662	0.0080
Average Monthly Salary for Beginning Teachers (log) X District Enrollment (log)	0.0010	0.0323	0.9760
Average Monthly Salary for Beginning Teachers (log) X % Free Lunch (non-high school)	-0.8002	0.2259	0.0000
Average Monthly Salary for Beginning Teachers (log) X % Limited English Proficient (non-high school)	0.9399	0.3714	0.0110
Average Monthly Salary for Beginning Teachers (log) X % Less Severe Special Education	-0.3475	1.1244	0.7570
Average Monthly Salary for Beginning Teachers (log) X % High school	-0.1551	1.1652	0.8940
Average Monthly Salary for Beginning Teachers (log) X Miles to Nearest Major Metro Center (log)	0.1362	0.0413	0.0010
Average Monthly Salary for Beginning Teachers (log) X % Severe Disability Special Education	-0.4318	2.3558	0.8550
Average Monthly Salary for Auxiliary Workers (log) X District Enrollment (log)	-0.0712	0.0237	0.0030
Average Monthly Salary for Auxiliary Workers (log) X % Free Lunch (non-high school)	0.1259	0.1406	0.3710

Table 2. The Cost Function Coefficients			
Variable	**Coefficient**	**Std. Error**	**P > [z]**
Average Monthly Salary for Auxiliary Workers (log) X % Limited English Proficient (non-high school)	0.0541	0.2355	0.8180
Average Monthly Salary for Auxiliary Workers (log) X % Less Severe Special Education	-0.3288	0.6727	0.6250
Average Monthly Salary for Auxiliary Workers (log) X % High school	0.0025	0.7219	0.9970
Average Monthly Salary for Auxiliary Workers (log) X Miles to Nearest Major Metro Center (log)	-0.0086	0.0261	0.7420
Average Monthly Salary for Auxiliary Workers (log) X % Severe Disability Special Education	-4.0026	1.7197	0.0200
District Enrollment (log) X % Free Lunch (non-high school)	-0.0283	0.0180	0.1150
District Enrollment (log) X % Limited English Proficient (non-high school)	-0.0073	0.0279	0.7940
District Enrollment (log) X % Less Severe Special Education	0.0517	0.1038	0.6180
District Enrollment (log) X % High school	-0.1106	0.1008	0.2730
District Enrollment (log) X Miles to Nearest Major Metro Center (log)	-0.0067	0.0029	0.0200
District Enrollment (log) X % Severe Disability Special Education	0.3093	0.2046	0.1310
% Free Lunch (non-high school) X % Limited English Proficient (non-high school)	-0.1629	0.1255	0.1950
% Free Lunch (non-high school) X % Less Severe Special Education	-0.9219	0.6392	0.1490
% Free Lunch (non-high school) X % High school	1.3131	0.6736	0.0510
% Free Lunch (non-high school) X Miles to Nearest Major Metro Center (log)	-0.0027	0.0185	0.8850
% Free Lunch (non-high school) X % Severe Disability Special Education	0.7504	1.3301	0.5730
% Limited English Proficient (non-high school) X % Less Severe Special Education	0.8380	1.0257	0.4140
% Limited English Proficient (non-high school) X % High school	2.1290	1.0860	0.0500
% Limited English Proficient (non-high school) X Miles to Nearest Major Metro Center (log)	-0.0211	0.0300	0.4820
% Limited English Proficient (non-high school) X % Severe Disability Special Education	-2.1749	2.6544	0.4130

Table 2. The Cost Function Coefficients			
Variable	**Coefficient**	**Std. Error**	**P > [z]**
% Less Severe Special Education X % High school	3.1454	3.0825	0.3080
% Less Severe Special Education X Miles to Nearest Major Metro Center (log)	0.1576	0.1065	0.1390
% Less Severe Special Education X % Severe Disability Special Education	-5.4724	6.5059	0.4000
% High school X Miles to Nearest Major Metro Center (log)	-0.0459	0.1234	0.7100
% High school X % Severe Disability Special Education	-22.2425	6.2576	0.0000
Miles to Nearest Major Metro Center (log) X % Severe Disability Special Education	-0.3553	0.2734	0.1940
Change in Average TAAS Passing Rate Squared	-1.3711	3.2738	0.6750
% Completing an Advanced Course, Squared	-0.5307	0.2852	0.0630
% Above Criterion on SAT/ACT, Squared	1.3219	0.3855	0.0010
Prior TAAS Passing Rate, Squared	0.3522	0.7959	0.6580
Average Monthly Salary for Beginning Teachers (log), Squared	-0.7100	0.2400	0.0030
Average Monthly Salary for Auxiliary Workers (log), Squared	0.0928	0.1237	0.4530
District Enrollment (log), Squared	0.0606	0.0165	0.0000
% Free Lunch (non-high school), Squared	0.2014	0.0763	0.0080
% Limited English Proficient (non-high school), Squared	-0.5946	0.1760	0.0010
% Less Severe Special Education, Squared	-0.4199	1.6588	0.8000
% High school, Squared	-2.4236	1.7029	0.1550
Miles to Nearest Major Metro Center (log), Squared	0.0051	0.0031	0.0970
% Severe Disability Special Education, Squared	-5.3537	5.0965	0.2940
District Enrollment (log), Cubed	-0.0017	0.0006	0.0070
School year 2000	0.0170	0.0059	0.0040
School year 2001	0.0408	0.0065	0.0000
School year 2002	0.0497	0.0071	0.0000
Number of Observations		2755	
Log Likelihood		2754.6451	
Wald Chi-Squared		3915.4800	
Prob > Chi-Squared		0.0000	

Table 3: Regression Sample Descriptive Statistics for 2002				
Variable	**Mean**	**Standard Deviation**	**Minimum**	**Maximum**
Operating Expenditures per Pupil (log)	8.61	0.15	8.24	9.39
Change in Average TAAS Passing Rate	0.03	0.02	-0.02	0.14
% Completing an Advanced Course	0.18	0.06	0.01	0.61
% Above Criterion on SAT/ACT	0.13	0.07	0.00	0.51
Prior TAAS Passing Rate	0.90	0.05	0.63	0.99
Average Monthly Salary for Beginning Teachers (log)	7.80	0.11	7.64	8.07
Average Monthly Salary for Auxiliary Workers (log)	7.09	0.11	6.84	7.52
Predicted Monthly Salary for Beginning Teachers (log)	7.79	0.05	7.69	7.93
Predicted Monthly Salary for Auxiliary Workers (log)	7.07	0.08	6.91	7.29
District Enrollment (log)	7.67	1.21	5.69	12.26
% Free Lunch (non-high school)	0.40	0.17	0.00	0.92
% Limited English Proficient (non-high school)	0.09	0.11	0.00	0.70
% Less Severe Special Education	0.10	0.03	0.04	0.24
% High school	0.29	0.03	0.20	0.43
Miles to Nearest Metro Center (log)	4.23	0.90	2.30	5.89
% Severe Disability Special Education	0.03	0.01	0.01	0.19
Operating Expenditures per Pupil	$6,441.82	$1,080.96	$4,393.08	$13,975.69
Average Monthly Salary for Beginning Teachers	$2,838.59	$310.26	$2,424.00	$3,697.63
Average Monthly Salary for Auxiliary Workers	$1,410.33	$165.20	$1,088.99	$2,136.19
Predicted Monthly Salary for Beginning Teachers	$2,809.16	$147.65	$2,528.03	$3,227.20
Predicted Monthly Salary for Auxiliary Workers	$1,367.87	$112.53	$1,162.40	$1,704.00
District Enrollment	5722.80	14196.88	295	210670
Miles to Nearest Metro Center	96.65	76.16	10	362

Table 4: Marginal Effects of the Baseline Model		
Variable	Marginal Effect	Standard Error
Change in Average TAAS Passing Rate	0.3354	0.1377
% Completing an Advanced Course	0.1646	0.0379
% Above Criterion on SAT/ACT	0.0716	0.0421
Prior TAAS Passing Rate	0.0353	0.0632
Average Monthly Salary for Beginning Teachers (log)	0.4201	0.0345
Average Monthly Salary for Auxiliary Workers (log)	0.2551	0.0193
District Enrollment (log)	-0.0809	0.0037
% Free Lunch (non-high school)	0.2816	0.0201
% Limited English Proficient (non-high school)	0.1915	0.0438
% Less Severe Special Education	0.5490	0.0823
% High school	0.5973	0.0810
Miles to Nearest Metro Center (log)	0.0208	0.0033
% Severe Disability Special Education	0.8253	0.2004
Log Likelihood		2754.65
Observations		2755

Table 5: The Distribution of Cost Efficiency							
Number of Districts	Mean	Standard Deviation	Minimum	25th percentile	Median	75th percentile	Maximum
694	0.9315	0.0341	0.7401	0.9168	0.9413	0.9546	0.9856

Table 6: Marginal Effects from Alternative Specifications					
Marginal Effect of:	A: Baseline Model	B: Model A Excluding Object Codes 6401-6499	C: Model A but with Free and Reduced Lunch	D: Model A but with Predicted Wages	E: "Cobb Douglas" Version of Model A
Change in Average TAAS Passing Rate	0.3354 (.1377)	0.3301 (.13737)	0.3033 (.1379)	0.4167 (.1450)	0.0076 (.1117)
% Completing an Advanced Course	0.1646 (.0379)	0.1437 (.0378)	0.1799 (.0379)	0.1209 (.0397)	0.0955 (.0325)
% Above Criterion on SAT/ACT	0.0716 (.0421)	0.0859 (.0419)	0.0512 (.0421)	0.1296 (.0449)	0.2515 (.0362)
Prior TAAS Passing Rate	0.0353 (.0632)	0.0210 (.0630)	0.0079 (.0629)	0.0872 (.0671)	-0.3010 (.0530)
Average Monthly Salary for Beginning Teachers (log)	0.4201 (.0345)	0.4145 (.0341)	0.4246 (.0345)	0.5995 (.0458)	0.4048 (.0316)
Average Monthly Salary for Auxiliary Workers (log)	0.2551 (.0193)	0.2540 (.0192)	0.2512 (.0193)	0.3051 (.0319)	0.2852 (.0196)
District Enrollment (log)	-0.0809 (.0037)	-0.0777 (.0037)	-0.0794 (.0037)	-0.0780 (.0047)	-0.0809 (.0037)
% Free Lunch (non-high school)	0.2816 (.0201)	0.2918 (.0201)	0.2475 (.0183)	0.2547 (.0215)	0.2048 (.0164)
% Limited English Proficient (non-high school)	0.1915 (.0438)	0.1857 (.0436)	0.2202 (.0433)	0.1987 (.0472)	0.1263 (.0223)
% Less Severe Special Education	0.5490 (.0823)	0.5502 (.0821)	0.5564 (.0824)	0.2907 (.0886)	0.6527 (.0759)
% High school	0.5973 (.0810)	0.5957 (.0807)	0.6212 (.0825)	0.5471 (.0868)	0.3981 (.0813)
Miles to Nearest Metro Center (log)	0.0208 (.0033)	0.0174 (.0033)	0.0228 (.0033)	0.0374 (.0046)	0.0267 (.0027)
% Severe Disability Special Education	0.8253 (.2004)	0.7897 (.1993)	0.8174 (.2009)	0.5923 (.2127)	-0.1805 (.1618)
Log Likelihood	2754.65	2768.07	2753.67	2568.25	2451.24
Observations	2755	2755	2755	2749	2755
Note: Standard errors in parentheses.					

Table 6: Marginal Effects from Alternative Specifications, continued					
Marginal Effect of:	A: Baseline Model	F: Model A but without frontier errors	G: Model A but with percent passing both math and reading	H: Model A with low income TAAS scores	I: Model A excluding all districts with less than 1600 students
Change in Average TAAS Passing Rate	0.3354 (.1377)	0.4180 (.1498)	0.1954 (.09509)	0.0763 (0.0940)	0.2435 (0.2289)
% Completing an Advanced Course	0.1646 (.0379)	0.1610 (.0407)	0.1592 (0.0379)	0.1958 (0.0442)	0.1486 (0.0681)
% Above Criterion on SAT/ACT	0.0716 (.0421)	0.0828 (.0453)	0.0744 (0.0432)	-0.0356 (0.0483)	-0.0312 (0.0718)
Prior TAAS Passing Rate	0.0353 (.0632)	0.0264 (.0680)	0.0263 (0.0481)	-0.0275 (0.0462)	0.1496 (0.1042)
Average Monthly Salary Beginning Teachers (log)	0.4201 (.0345)	0.4173 (.0381)	0.4112 (0.0345)	0.4074 (0.0385)	0.4002 (0.0524)
Average Monthly Salary Auxiliary Workers (log)	0.2551 (.0193)	0.2800 (.0208)	0.2635 (0.0192)	0.2933 (0.0225)	0.3291 (0.0326)
District Enrollment (log)	-0.0809 (.0037)	-0.0803 (.0042)	-0.0811 (0.0038)	-0.0814 (0.0043)	-0.0676 (0.0107)
% Free Lunch (non-high school)	0.2816 (.0201)	0.2690 (.0210)	0.2736 (0.0200)	0.2657 (0.0227)	0.3364 (0.0317)
% Limited English Proficient (non-high school)	0.1915 (.0438)	0.1942 (.0471)	0.1963 (0.0438)	0.1615 (0.0474)	0.1644 (0.0629)
% Less Severe Special Education	0.5490 (.0823)	0.5469 (.0914)	0.4968 (0.0819)	0.5001 (0.0959)	0.6533 (0.1527)
% High school	0.5973 (.0810)	0.6181 (.0890)	0.5929 (0.0808)	0.7429 (0.0955)	0.7688 (0.1553)
Miles to Nearest Metro Center (log)	0.0208 (.0033)	0.0238 (.0035)	0.0204 (0.0033)	0.0194 (0.0037)	0.0100 (0.0054)
% Severe Disability Special Education	0.8253 (.2004)	0.7594 (.2148)	0.8667 (0.2005)	0.1618 (0.2440)	0.5882 (0.3841)
Log Likelihood	2754.6	2644.6	2754.3	2348.1	1745.0
Observations	2755	2755	2755	2179	1418
Note: Standard errors in parentheses.					

Table 6: Marginal Effects from Alternative Specifications, continued				
Marginal Effect of:	J: Model A passing both math & reading, free & reduced	K: Model J using 2004 data	L. Model K excluding Advanced Courses & SAT/ACT	M: Model L including completion rate
Change in Average Percent Passing Both Math & Reading (TAAS or TAKS)	0.1774 (0.0953)	0.0913 (0.1186)	0.1376 (0.1181)	0.0903 (0.1220
Completion Rate				-0.1615 (0.2172
% Completing an Advanced Course	0.1744 (0.0380)	0.2508 (0.0657)		
% Above Criterion on SAT/ACT	0.0530 (0.0432)	-0.0704 (0.0761)		
Prior Passing Rate (TAAS or TAKS)	0.0091 (0.0479)	0.0116 (0.0558)	0.0215 (0.0660)	0.0197 (0.0752
Average Monthly Salary Beginning Teachers (log)	0.4149 (0.0345)	0.3672 (0.0641)	0.3869 (0.0613)	0.4070 (0.0761
Average Monthly Salary Auxiliary Workers (log)	0.2595 (0.0193)	0.1844 (0.0387)	0.2247 (0.0438)	0.2189 (0.0420
District Enrollment (log)	-0.0795 (0.0032)	-0.0736 (0.0074)	-0.0809 (0.0077)	-0.0862 (0.0082
% Free and Reduced Lunch (non-high school)	0.2401 (0.0182)	0.1831 (0.0361)	0.1790 (0.0410)	0.1682 (0.0440
% Limited English Proficient (non-high school)	0.2253 (0.0433)	0.2572 (0.0842)	0.2928 (0.0814)	0.2853 (0.0846
% Less Severe Special Education	0.5056 (0.0822)	0.8037 (0.1648)	0.7251 (0.1781)	0.7256 (0.1971
% High school	0.6159 (0.0808)	0.2326 (0.1664)	0.3676 (0.1528)	0.2871 (0.1425
Miles to Nearest Metro Center (log)	0.0222 (0.0033)	0.0280 (0.0066)	0.0286 (0.0073)	0.0296 (0.0069
% Severe Disability Special Education	0.8598 (0.2011)	1.3051 (0.4192)	1.5847 (0.4681)	1.5997 (0.5099
Log Likelihood	2752.70	713.94	691.51	694.00
Observations	2755	714	726	726
Note: Standard errors in parentheses.				

Table 7. Distribution of Predicted Expenditures (in 2004$)*

Alternative Models	Correlation with predicted expenditures from Model A	Mean Predicted Expenditures	Standard Deviation of Predicted Expenditures	Minimum Predicted Expenditures	Maximum Predicted Expenditures
Model A	1.0000	6,283.06	561.94	5,252.07	14,832.45
Model B	0.9952	6,168.64	530.97	5,161.60	14,291.06
Model C	0.9971	6,270.18	562.42	5,210.97	14,154.71
Model D	0.8446	6,014.69	703.80	5,026.33	14,889.53
Model E	0.9718	6,336.16	538.30	5,391.94	13,534.03
Model F	0.9978	6,335.48	553.73	5,318.52	15,105.36
Model G	0.9998	6,294.06	551.27	5,281.84	14,749.30
Model H	0.9903	6,282.11	555.36	5,237.39	14,190.90
Model I	0.9364	6,015.34	552.60	5,047.81	11,730.79
Model J	0.9967	6,290.31	548.89	5,247.73	14,070.87
Model K	0.9428	6,477.51	505.07	5,419.63	16,077.38
Model L	0.9562	6,381.04	537.24	5,306.30	17,684.42
Model M	0.9512	6,444.52	538.67	5,346.61	17,531.71

*The predicted cost of average performance on the outcome measures in the model, weighted by 2004 enrollment.

Table 8. The Predicted Cost of Achieving TAKS Performance Levels (in 2004$)*					
Alternative Models	Correlation with Model A at 55-Percent	Mean	Standard Deviation	Minimum	Maximum
55-Percent Passing Rate					
Model A	1.0000	6,389.23	617.00	5,252.07	14,832.45
Model J	0.9930	6,410.25	601.60	5,247.73	14,283.39
Model K	0.9337	6,483.18	507.72	5,419.63	16,077.38
Model L	0.9340	6,395.35	539.80	5,309.24	17,664.62
70-Percent Passing Rate					
Model K	0.9413	6,522.59	524.15	5,419.63	16,126.76
Model L	0.9497	6,453.91	561.41	5,309.24	17,746.49
80-Percent Passing Rate					
Model K	0.9420	6,568.66	539.24	5,423.78	16,274.65
Model L	0.9518	6,522.80	583.67	5,315.38	17,992.40
Actual Expenditures					
2002	0.4638	6,561.32	738.10	4,289.64	20,649.96
2003	0.5118	6,725.35	738.37	4,729.72	19,568.80
2004	0.4944	6,513.05	747.95	4,598.84	22,378.29

*The predicted cost of performance on the outcome measures in the model, weighted by 2004 enrollment. Actual expenditures for 2004 are imputed.

Figure 1: Predicted cost distribution from full translog model and from the linearized model that includes the auxiliary-enrollment interaction adjustment. The correlation between the linearized and the full translog models is 0.932.

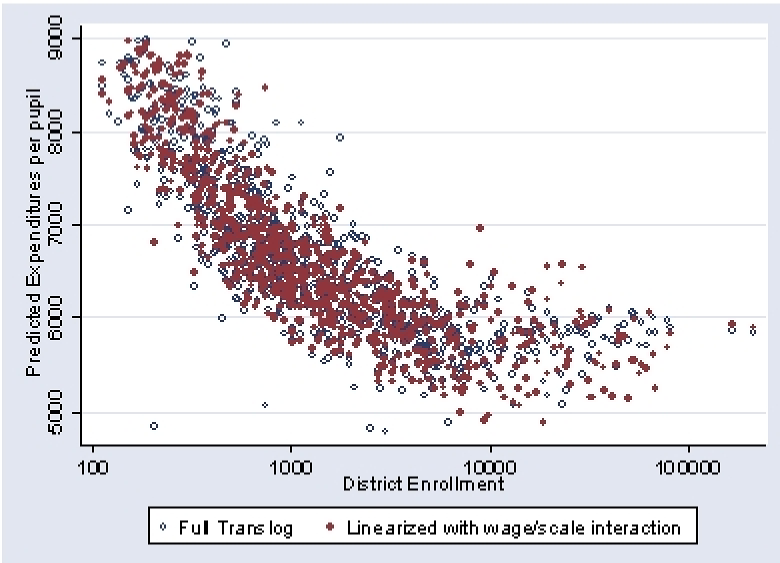


Figure 2: Predicted cost distribution from full translog model and linearized model that does not include the auxiliary-enrollment interaction adjustment. The correlation between the linearized and the full translog models is .937.

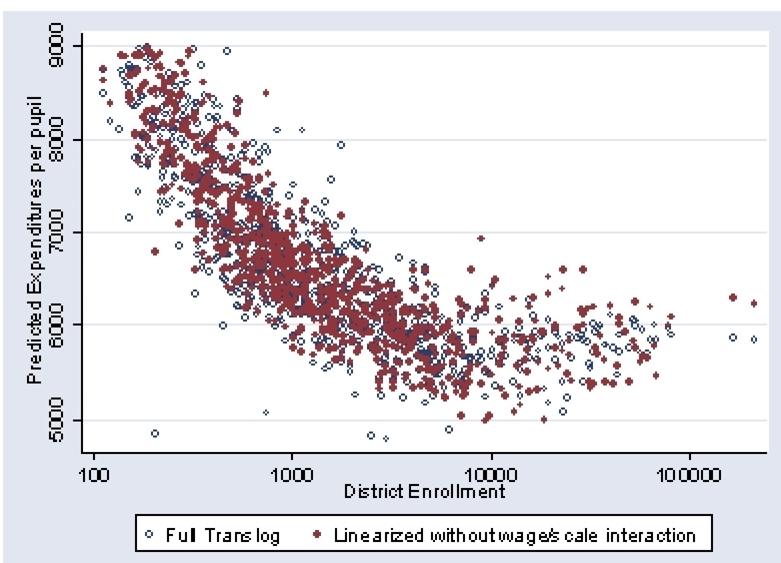


Figure 3a. Comparing Predicted Costs from Models J and K

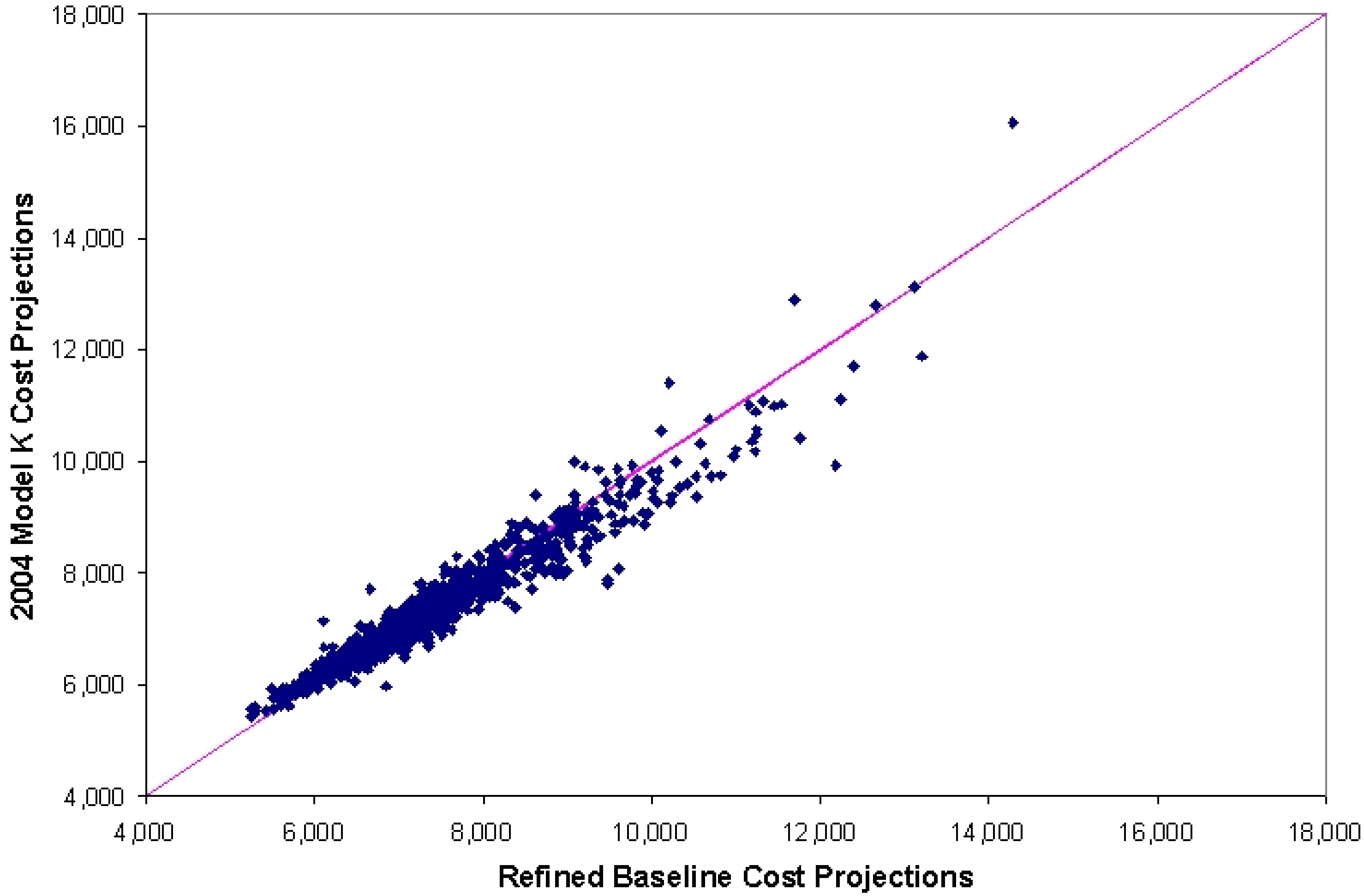


Figure 3b. Comparing Predicted Costs from Models J and L

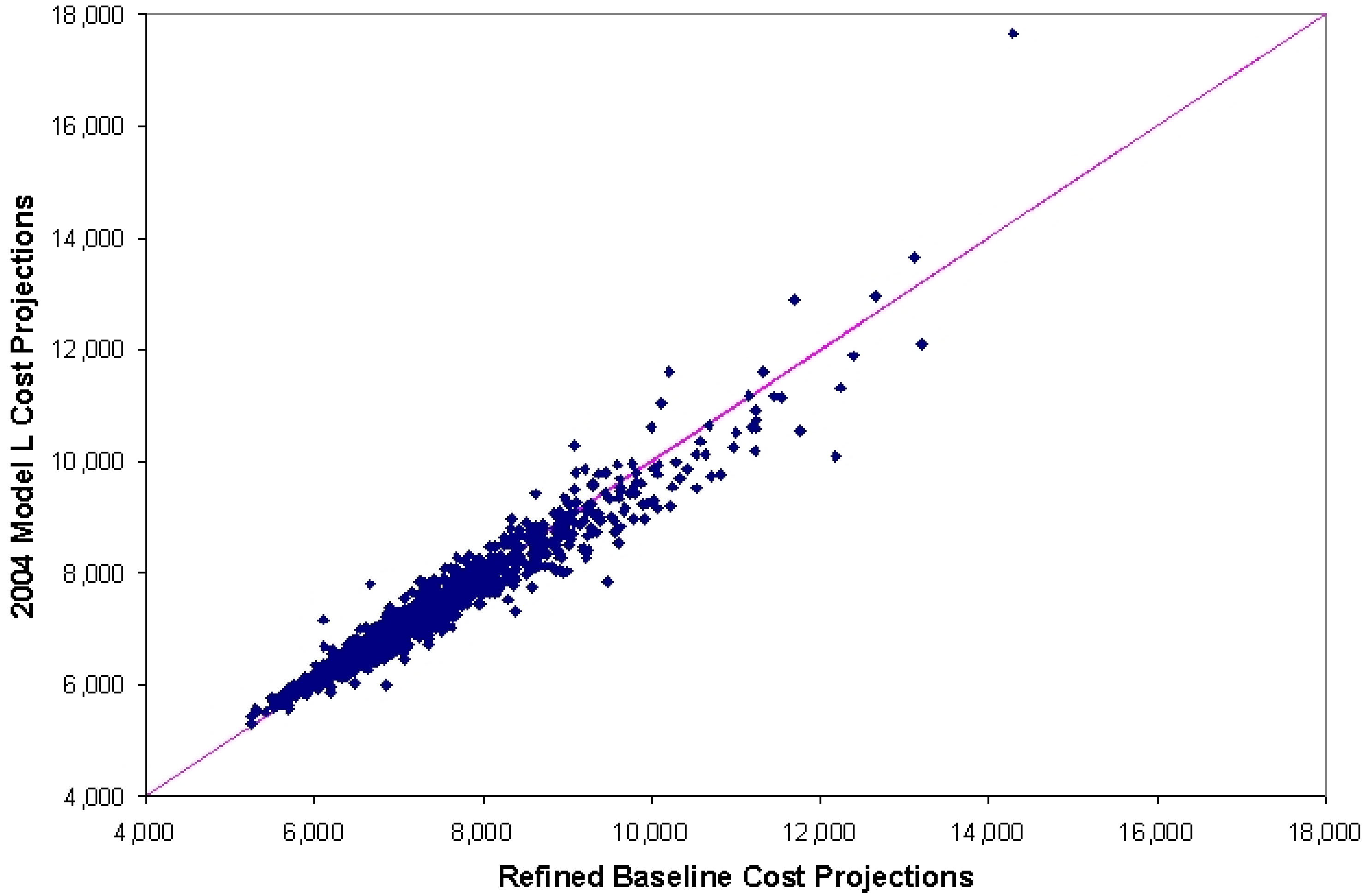


Figure 4a. Comparing Predicted Costs from Models J and K, by enrollment

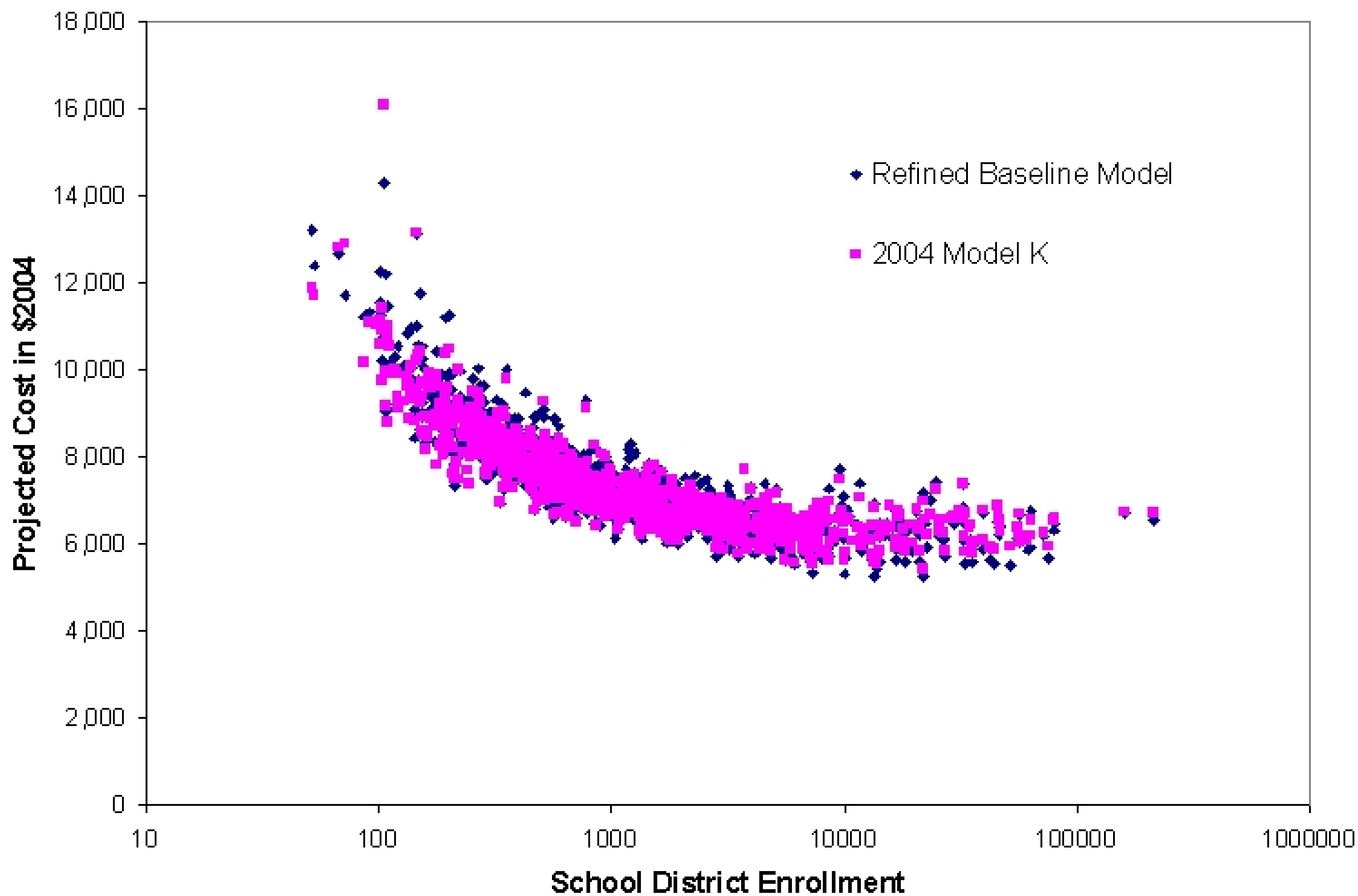
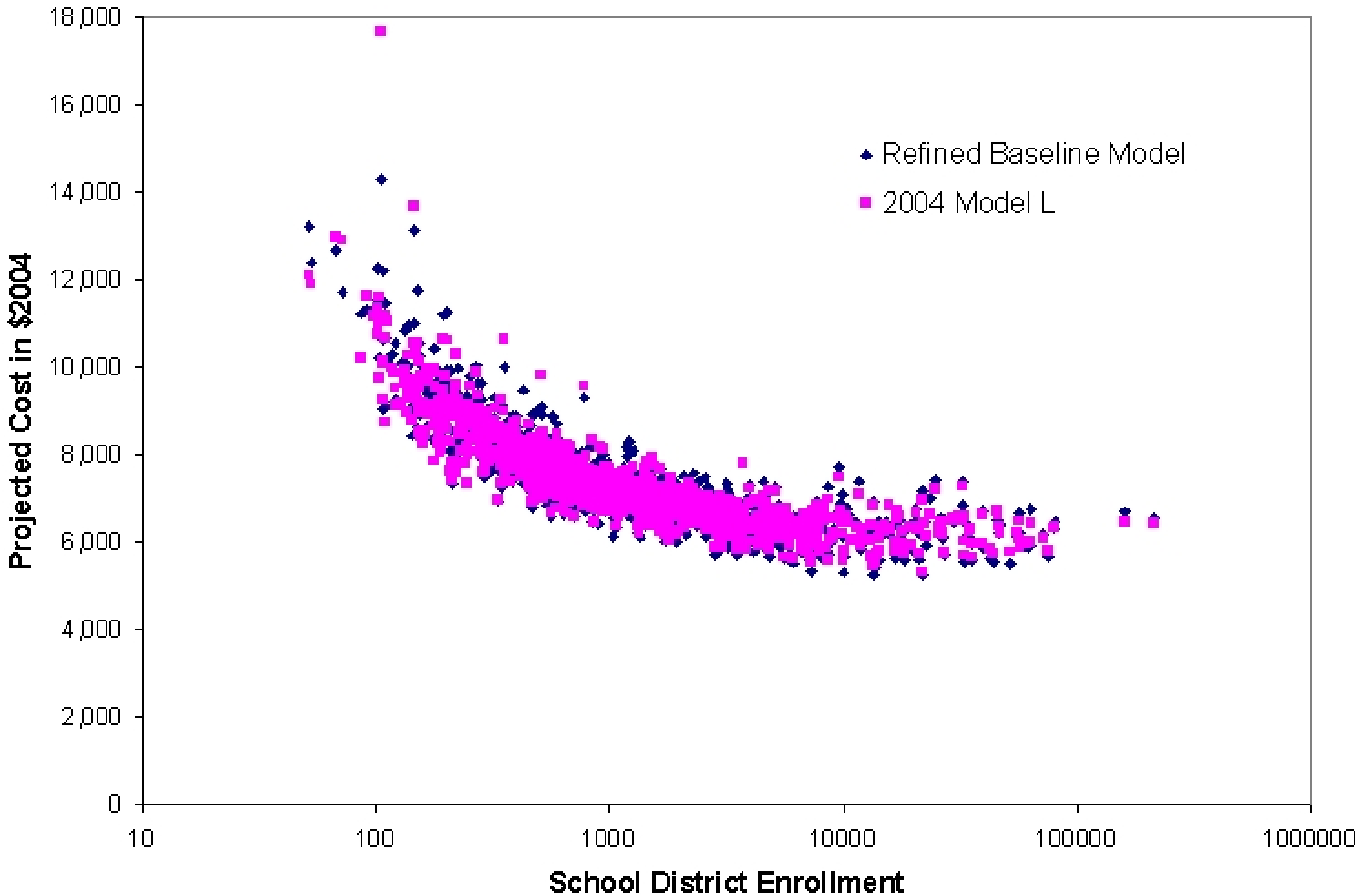


Figure 4b:. Comparing Predicted Costs from Models J and L, by enrollment



Appendix A. Stochastic Frontier Modeling, Heteroskedasticity and Cost Efficiency

Stochastic production frontier models were introduced by Aigner, Lovell, and Schmidt (1977). Kumbhakar and Lovell (2000) provide a good introduction to this area.

Suppose that a school district has a cost function $f(q_i, p_i, \beta)$, where q represents the district output and p represents district input quantities or input prices. In a world without error or inefficiency, the ith district costs would be

$$c_i = f(q_i, p_i, \beta)$$

A fundamental element of stochastic frontier analysis is that each district might have costs greater than those specified by the cost function due to a degree of inefficiency. Specifically,

$$c_i = f(q_i, p_i, \beta)\xi_i$$

where ξ_i is the level of inefficiency for district i. If $\xi_i = 1$, then the firm is achieving the optimal level of costs for achieving the specified output level using the specified inputs and the implied technology. When $\xi_i > 1$, the firm has costs that are greater than necessary.

Costs are also assumed to be subject to random shocks, implying that

$$c_i = f(q_i, p_i, \beta)\xi_i \exp(\upsilon_i)$$

Taking the natural log of both sides yields

$$\ln(q_i) = \ln\{f(z_i, \beta)\} + \ln(\xi_i) + \upsilon_i$$

Defining $u_i = \ln(\xi_i)$, and noting that $\xi_i \geq 1$, we have:

$$\ln(c_i) = \ln\{f(q_i, p_i, \beta)\} + u_i + v_i$$

where $u_i \geq 0$.

The stochastic frontier version thus modifies the assumption about the error term to have it consisting of two components, a standard two-sided term to capture random un-modeled differences across districts and a one-sided term to capture inefficiencies measured as distance to the cost frontier. Intuitively, the inefficiency effect raises expenditure.

Different specifications of the u_i and υ_i terms give rise to distinct models. In all models, the idiosyncratic component υ_i is assumed to be independently $N(0,\sigma_\upsilon)$ distributed over the observations. The basic models differ in their specification of the inefficiency term, u_i. We assume that the u_i are independently half-normally distributed $N^+(0, \sigma_u^2)$ and estimate the relationship using maximum likelihood.

We have estimators for the parameters of models when the error components are conditionally heteroskedastic, so these models only assume independence.

Heteroskedastcity

In many cases, the error terms may not have constant variance. Heteroskedasticity can be modeled in either error term as a linear function of a set of covariates. The variance of either the technical inefficiency or the idiosyncratic component may be modeled as

$$\sigma_i^2 = \exp(w_i \delta)$$

It is possible to simultaneously specify covariates for both σ_{u_i} and σ_{v_i}.

[Technical Note: The frontier maximizes the log-likelihood function of a stochastic frontier model using the Newton-Raphson method, and the estimated variance-covariance matrix is calculated as the inverse of the negative Hessian (matrix of second partial derivatives).]

Consider an equation of the form

$$y_i = x_i\beta + v_i + u_i$$

where

y_i is the dependent variable,

x_i is an $1 \times k$ vector of observations on the independent variables included as covariates,

β is a $k \times 1$ vector of coefficients, and

The log-likelihood function is

$$\ln L = \sum_{i=1}^{N} \left\{ \frac{1}{2}\ln\left(\frac{2}{\pi}\right) - \ln \sigma_S + \ln \Phi\left(+ \frac{\varepsilon_i \lambda}{\sigma_S}\right) - \frac{\varepsilon_i^2}{2\sigma_S^2} \right\}$$

where $\sigma_S = \left(\sigma_u^2 + \sigma_v^2\right)^{1/2}$, $\lambda = \sigma_u / \sigma_v$, $\gamma = \sigma_u^2 / \sigma_S^2$, $\varepsilon_i = y_i - x_i\beta$, and $\Phi(\ )$ is the cumulative distribution function of the standard normal distribution.

To obtain estimation for u_i, the mean of the conditional distribution $f(u| \in)$ can be used.

$$E(u_i|\varepsilon_i) = \mu_{*i} + \sigma_* \left\{ \frac{\phi(-\mu_{*i}/\sigma_*)}{1 - \Phi(\mu_{*i}/\sigma_*)} \right\}$$

Then, estimates of cost efficiency will be estimated by

$$\begin{aligned} E_i &= E\{\exp(+u_i)|\varepsilon_i\} \\ &= \left\{ \frac{1 - \Phi(\sigma_* - \mu_{*i}/\sigma_*)}{1 - \Phi(-\mu_{*i}/\sigma_*)} \right\} \exp\left(-\mu_{*i} + \frac{1}{2}\sigma_*^2\right) \end{aligned}$$

where μ_i are defined for the normal/half-normal model is

$$\mu_{*i} = \varepsilon_i \, \sigma_u^2 / \sigma^2$$

$$\sigma_* = \sigma_u \sigma_v / \sigma_S$$

In the half-normal, when heteroskedasticity is assumed, the standard deviations, σ_u or σ_v, will be replaced in the above equations by

$$\sigma_i^2 = \exp(w_i \delta)$$

where *w* is the vector of explanatory variables in the variance function.

Cost Efficiency Measurement

As argued in Kumbhakar and Lovell (138), with the cost frontier specified as being stochastic, an appropriate measure of cost efficiency is

$$CE_i = \frac{c(q_i, p_i; \beta) \cdot \exp\{v_i\}}{E_i}$$

which defines cost efficiency as the ratio of minimum cost attainable in an environment characterized by $\exp\{v_i\}$ to observed expenditure. $CE_i \leq 1$, with $CE_i = 1$ if, and only if,

$E_i = c(q_i, p_i; \beta) \cdot \exp\{v_i\}$.

Under the natural logarithmic transformation to the cost function,

$\ln CE_i = -u_i$

and the cost efficiency measure can be calculated as

$CE_i = \exp\{-u_i\}$.

The regression provides estimates of the compound error $\varepsilon_i = v_i + u_i$, not of u_i. To extract the relevant information on u_i from the estimates of ε_i, we use the mean of the conditional distribution of $\exp\{-u_i\}$ given, $E(\exp\{-u_i|\varepsilon_i\})$, as point estimator for cost efficiency (see Battese and Coelli (1988)).

Appendix B

$$Var\left(me\left(q_1\right)\right) = Var\left(\tilde{a}_1^2\right) + 2\bar{q}_2 Cov\left(\tilde{a}_1\tilde{d}_{12}\right) + \bar{q}_2^2 Var\left(\tilde{d}_{12}^2\right) + 2\bar{q}_3 Cov\left(\tilde{a}_1\tilde{d}_{13}\right) + 2\bar{q}_2\bar{q}_3 Cov\left(\tilde{d}_{12}\tilde{d}_{13}\right)$$

$$+ \bar{q}_3^2 Var\left(\tilde{d}_{13}^2\right) + 2\bar{w}_1 Cov\left(\tilde{a}_1\tilde{e}_{11}\right) + 2\bar{q}_2\bar{w}_1 Cov\left(\tilde{d}_{12}\tilde{e}_{11}\right) + 2\bar{q}_3\bar{w}_1 Cov\left(\tilde{d}_{13}\tilde{e}_{11}\right) + \bar{w}_1^2 Var\left(\tilde{e}_{11}^2\right)$$

$$+ 2\bar{w}_2 Cov\left(\tilde{a}_1\tilde{e}_{12}\right) + 2\bar{q}_2\bar{w}_2 Cov\left(\tilde{d}_{12}\tilde{e}_{12}\right) + 2\bar{q}_3\bar{w}_2 Cov\left(\tilde{d}_{13}\tilde{e}_{12}\right) + 2\bar{w}_1\bar{w}_2 Cov\left(\tilde{e}_{11}\tilde{e}_{12}\right) + \bar{w}_2^2 Var\left(\tilde{e}_{12}^2\right)$$

$$+ 2\bar{x}_1 Cov\left(\tilde{a}_1\tilde{g}_{11}\right) + 2\bar{q}_2\bar{x}_1 Cov\left(\tilde{d}_{12}\tilde{g}_{11}\right) + 2\bar{q}_3\bar{x}_1 Cov\left(\tilde{d}_{13}\tilde{g}_{11}\right) + 2\bar{w}_1\bar{x}_1 Cov\left(\tilde{e}_{11}\tilde{g}_{11}\right) + 2\bar{w}_2\bar{x}_1 Cov\left(\tilde{e}_{12}\tilde{g}_{11}\right)$$

$$+ \bar{x}_1^2 Var\left(\tilde{g}_{11}^2\right) + 2\bar{x}_2 Cov\left(\tilde{a}_1\tilde{g}_{12}\right) + 2\bar{q}_2\bar{x}_2 Cov\left(\tilde{d}_{12}\tilde{g}_{12}\right) + 2\bar{q}_3\bar{x}_2 Cov\left(\tilde{d}_{13}\tilde{g}_{12}\right) + 2\bar{w}_1\bar{x}_2 Cov\left(\tilde{e}_{11}\tilde{g}_{12}\right)$$

$$+ 2\bar{w}_2\bar{x}_2 Cov\left(\tilde{e}_{12}\tilde{g}_{12}\right) + 2\bar{x}_1\bar{x}_2 Cov\left(\tilde{g}_{11}\tilde{g}_{12}\right) + \bar{x}_2^2 Var\left(\tilde{g}_{12}^2\right) + 2\bar{x}_3 Cov\left(\tilde{a}_1\tilde{g}_{13}\right) + 2\bar{q}_2\bar{x}_3 Cov\left(\tilde{d}_{12}\tilde{g}_{13}\right)$$

$$+ 2\bar{q}_3\bar{x}_3 Cov\left(\tilde{d}_{13}\tilde{g}_{13}\right) + 2\bar{w}_1\bar{x}_3 Cov\left(\tilde{e}_{11}\tilde{g}_{13}\right) + 2\bar{w}_2\bar{x}_3 Cov\left(\tilde{e}_{12}\tilde{g}_{13}\right) + 2\bar{x}_1\bar{x}_3 Cov\left(\tilde{g}_{11}\tilde{g}_{13}\right)$$

$$+ 2\bar{x}_2\bar{x}_3 Cov\left(\tilde{g}_{12}\tilde{g}_{13}\right) + \bar{x}_3^2 Var\left(\tilde{g}_{13}^2\right) + 2\bar{x}_4 Cov\left(\tilde{a}_1\tilde{g}_{14}\right) + 2\bar{q}_2\bar{x}_4 Cov\left(\tilde{d}_{12}\tilde{g}_{14}\right) + 2\bar{q}_3\bar{x}_4 Cov\left(\tilde{d}_{13}\tilde{g}_{14}\right)$$

$$+ 2\bar{w}_1\bar{x}_4 Cov\left(\tilde{e}_{11}\tilde{g}_{14}\right) + 2\bar{w}_2\bar{x}_4 Cov\left(\tilde{e}_{12}\tilde{g}_{14}\right) + 2\bar{x}_1\bar{x}_4 Cov\left(\tilde{g}_{11}\tilde{g}_{14}\right) + 2\bar{x}_2\bar{x}_4 Cov\left(\tilde{g}_{12}\tilde{g}_{14}\right)$$

$$+ 2\bar{x}_3\bar{x}_4 Cov\left(\tilde{g}_{13}\tilde{g}_{14}\right) + \bar{x}_4^2 Var\left(\tilde{g}_{14}^2\right) + 2\bar{x}_5 Cov\left(\tilde{a}_1\tilde{g}_{15}\right) + 2\bar{q}_2\bar{x}_5 Cov\left(\tilde{d}_{12}\tilde{g}_{15}\right) + 2\bar{q}_3\bar{x}_5 Cov\left(\tilde{d}_{13}\tilde{g}_{15}\right)$$

$$+ 2\bar{w}_1\bar{x}_5 Cov\left(\tilde{e}_{11}\tilde{g}_{15}\right) + 2\bar{w}_2\bar{x}_5 Cov\left(\tilde{e}_{12}\tilde{g}_{15}\right) + 2\bar{x}_1\bar{x}_5 Cov\left(\tilde{g}_{11}\tilde{g}_{15}\right) + 2\bar{x}_2\bar{x}_5 Cov\left(\tilde{g}_{12}\tilde{g}_{15}\right)$$

$$+ 2\bar{x}_3\bar{x}_5 Cov\left(\tilde{g}_{13}\tilde{g}_{15}\right) + 2\bar{x}_4\bar{x}_5 Cov\left(\tilde{g}_{14}\tilde{g}_{15}\right) + \bar{x}_5^2 Var\left(\tilde{g}_{15}^2\right) + 2\bar{x}_6 Cov\left(\tilde{a}_1\tilde{g}_{16}\right) + 2\bar{q}_2\bar{x}_6 Cov\left(\tilde{d}_{12}\tilde{g}_{16}\right)$$

$$+ 2\bar{q}_3\bar{x}_6 Cov\left(\tilde{d}_{13}\tilde{g}_{16}\right) + 2\bar{w}_1\bar{x}_6 Cov\left(\tilde{e}_{11}\tilde{g}_{16}\right) + 2\bar{w}_2\bar{x}_6 Cov\left(\tilde{e}_{12}\tilde{g}_{16}\right) + 2\bar{x}_1\bar{x}_6 Cov\left(\tilde{g}_{11}\tilde{g}_{16}\right)$$

$$+ 2\bar{x}_2\bar{x}_6 Cov\left(\tilde{g}_{12}\tilde{g}_{16}\right) + 2\bar{x}_3\bar{x}_6 Cov\left(\tilde{g}_{13}\tilde{g}_{16}\right) + 2\bar{x}_4\bar{x}_6 Cov\left(\tilde{g}_{14}\tilde{g}_{16}\right) + 2\bar{x}_5\bar{x}_6 Cov\left(\tilde{g}_{15}\tilde{g}_{16}\right) + \bar{x}_6^2 Var\left(\tilde{g}_{16}^2\right)$$

$$+ 2\bar{x}_7 Cov\left(\tilde{a}_1\tilde{g}_{17}\right) + 2\bar{q}_2\bar{x}_7 Cov\left(\tilde{d}_{12}\tilde{g}_{17}\right) + 2\bar{q}_3\bar{x}_7 Cov\left(\tilde{d}_{13}\tilde{g}_{17}\right) + 2\bar{w}_1\bar{x}_7 Cov\left(\tilde{e}_{11}\tilde{g}_{17}\right)$$

$$+ 2\bar{w}_2\bar{x}_7 Cov\left(\tilde{e}_{12}\tilde{g}_{17}\right) + 2\bar{x}_1\bar{x}_7 Cov\left(\tilde{g}_{11}\tilde{g}_{17}\right) + 2\bar{x}_2\bar{x}_7 Cov\left(\tilde{g}_{12}\tilde{g}_{17}\right) + 2\bar{x}_3\bar{x}_7 Cov\left(\tilde{g}_{13}\tilde{g}_{17}\right)$$

$$+ 2\bar{x}_4\bar{x}_7 Cov\left(\tilde{g}_{14}\tilde{g}_{17}\right) + 2\bar{x}_5\bar{x}_7 Cov\left(\tilde{g}_{15}\tilde{g}_{17}\right) + 2\bar{x}_6\bar{x}_7 Cov\left(\tilde{g}_{16}\tilde{g}_{17}\right) + \bar{x}_7^2 Var\left(\tilde{g}_{17}^2\right) + 2\bar{x}_8 Cov\left(\tilde{a}_1\tilde{g}_{18}\right)$$

$$+ 2\bar{q}_2\bar{x}_8 Cov\left(\tilde{d}_{12}\tilde{g}_{18}\right) + 2\bar{q}_3\bar{x}_8 Cov\left(\tilde{d}_{13}\tilde{g}_{18}\right) + 2\bar{w}_1\bar{x}_8 Cov\left(\tilde{e}_{11}\tilde{g}_{18}\right) + 2\bar{w}_2\bar{x}_8 Cov\left(\tilde{e}_{12}\tilde{g}_{18}\right)$$

$$+ 2\bar{x}_1\bar{x}_8 Cov\left(\tilde{g}_{11}\tilde{g}_{18}\right) + 2\bar{x}_2\bar{x}_8 Cov\left(\tilde{g}_{12}\tilde{g}_{18}\right) + 2\bar{x}_3\bar{x}_8 Cov\left(\tilde{g}_{13}\tilde{g}_{18}\right) + 2\bar{x}_4\bar{x}_8 Cov\left(\tilde{g}_{14}\tilde{g}_{18}\right)$$

$$+ 2\bar{x}_5\bar{x}_8 Cov\left(\tilde{g}_{15}\tilde{g}_{18}\right) + 2\bar{x}_6\bar{x}_8 Cov\left(\tilde{g}_{16}\tilde{g}_{18}\right) + 2\bar{x}_7\bar{x}_8 Cov\left(\tilde{g}_{17}\tilde{g}_{18}\right) + \bar{x}_8^2 Var\left(\tilde{g}_{18}^2\right) + 4\bar{q}_1 Cov\left(\tilde{a}_1\tilde{d}_{11}\right)$$

$$+ 4\bar{q}_2\bar{q}_1 Cov\left(\tilde{d}_{12}\tilde{d}_{11}\right) + 4\bar{q}_3\bar{q}_1 Cov\left(\tilde{d}_{13}\tilde{d}_{11}\right) + 4\bar{w}_1\bar{q}_1 Cov\left(\tilde{e}_{11}\tilde{d}_{11}\right) + 4\bar{w}_2\bar{q}_1 Cov\left(\tilde{e}_{12}\tilde{d}_{11}\right)$$

$$+ 4\bar{x}_1\bar{q}_1 Cov\left(\tilde{g}_{11}\tilde{d}_{11}\right) + 4\bar{x}_2\bar{q}_1 Cov\left(\tilde{g}_{12}\tilde{d}_{11}\right) + 4\bar{x}_3\bar{q}_1 Cov\left(\tilde{g}_{13}\tilde{d}_{11}\right) + 4\bar{x}_4\bar{q}_1 Cov\left(\tilde{g}_{14}\tilde{d}_{11}\right)$$

$$+ 4\bar{x}_5\bar{q}_1 Cov\left(\tilde{g}_{15}\tilde{d}_{11}\right) + 4\bar{x}_6\bar{q}_1 Cov\left(\tilde{g}_{16}\tilde{d}_{11}\right) + 4\bar{x}_7\bar{q}_1 Cov\left(\tilde{g}_{17}\tilde{d}_{11}\right) + 4\bar{x}_8\bar{q}_1 Cov\left(\tilde{g}_{18}\tilde{d}_{11}\right) + 4\bar{q}_1^2 Var\left(\tilde{d}_{11}^2\right)$$

Appendix C. Definitions of variables

Operating expenditures. From the TEA actual expenditure files, the sum of all expenditures with object codes of 6100-6499 and function codes of 11–53, excluding functions 34 and 35. The March 2004 report inadvertently excluded object codes 6401–6499. This supplement includes those expenditures.

Operating expenditures per pupil. Operating expenditures divided by total district enrollment.

Change in average TAAS passing rate. The three-year average of 5–8th and 10th grade math and reading passing rates on the TAAS test, minus the three-year average of 3–6th and 8th grade math and reading passing rates for the same students two years earlier. For a district in 2002, the average math passing rate for 5th graders in that district in 2002—ignoring students with a missing math Texas Learning Index (TLI) score, students we cannot match back to a non-missing math TLI score in 2000, students who did not advance two grades between 2000 and 2002, and students whose ID number appears more than once in the 2002 student-level TAAS data—is added to the average math passing rate for 6th, 7th, 8th and 10th graders, and also to the average reading passing rate for 5th –8th and 10th graders in 2002. The one-year average 5–8th and 10th grade math and reading passing rate for this district would be the simple average of these 10 grade-test passing rate combinations. If any of these grade-test combinations had less than 10 scored students then the whole passing rate was set to missing. Next we calculated the same one-year average 5–8th and 10th grade math and reading passing rate for that same district in year 2001, and again in year 2000. The three-year average 5–8th and 10th grade math and reading passing rate for that district is the simple average of these three one-year average passing rates. For this same student population, the three-year average of 3–6th and 8th grade math and reading passing rates from two years prior is calculated the same way, as the simple average of the one-year passing rates for 1998, 1999 and 2000 for those grades. This three-year average of 3–6th grade math and reading passing rates from two years prior for the matched-students sample is the lagged TAAS measure in the cost function. The TAAS output measure is then the difference between these two three year averages, one for 2000-2002 on 5–8th and 10th grades, the other for the same students in years 1998–2000 and grades 3–6 and 8. The raw data for these calculations come from student files provided by TEA.

Percent completing an advanced course. The three-year average of the percent of 9-12th grade students in a district who complete at least one advanced course. Data from TEA's Academic Excellence Indicator System (AEIS).

Percent above criterion on SAT/ACT. The three-year average of the product of two measures: 1) the percent of graduates who took either college admissions test, and 2) the percent of examinees who scored at or above the criterion score on either test (1,110 on the SAT, or 24 on the ACT). Data from AEIS.

Completion rate. The completion rate is the number of graduates, GED recipients, and continuers expressed as a percentage of total students in their class. Data from the 2004 Texas Accountability Rating System (http://www.tea.state.tx.us/perfreport/account/2004/index.html).

Average monthly salary for beginning teachers. This variable is average monthly teacher salary for teachers with two years experience or less, adjusted to 1998 dollars using the Employment Cost Index described above. To find average monthly salary, we calculated average daily salary as base salary (payroll activity code 80) divided by effective days worked. (Effective days worked is days employed multiplied by percent worked per day.) Then we multiplied average daily salary by 18.7 to reflect the 187 days in a standard 10-month teaching contract. For individuals who worked on more than one campus or otherwise held more than one position, average monthly salary was the weighted average of the average monthly salary in each position using effective days in the position as the weights. Teachers whose records indicated that they worked less than 93.5 days or more than 261 days were excluded. For individuals who received a separate incentive that exceeded $100 per year for teaching in a bilingual/ESL program (payroll code 15), average monthly ESL supplement was added to average monthly salary. Average monthly ESL supplement was total payments under payroll code 15 divided by effective days worked and then multiplied by 18.7. The average monthly ESL supplement was included in total average monthly salary because it is the only supplement specifically designated for teaching duties. All other supplements were excluded. Data come from PEIMS personnel files.

Average monthly salary for auxiliary workers. This variable is the average monthly salary of school employees who are not teachers, administrators or support staff, adjusted to 1998 dollars. To find average monthly salary, we calculated average daily salary as base salary (payroll activity code 80) divided by effective days worked. (Effective days worked is days employed multiplied by percent worked per day.) Then we multiplied average daily salary by 18.7 to reflect the 187 days in a standard 10-month teaching contract. Individuals who worked fewer than 93.5 days or more than 261 days were excluded. The data come from the PEIMS personnel files.

Prior TAAS passing rate. To at least partly control for a district's student body ability level, we include the two-year lagged scores from the student-level data for the same students that generated the average passing rates that were used to generate average passing rate changes. Thus, as stated above, the three-year average of 3–6th grade math and reading passing rates from two years prior for the matched-students sample is the lagged TAAS measure in the cost function. Districts that lack a prior test score are not included in the estimation, but may be included in the cost projections. When predicting costs, if a district was missing a prior TAAS passing rate, then that variable was given the same value as the benchmark district.

Enrollment. Data on district enrollment come from the AEIS files.

Percent free lunch, non-high school. The percentage of students enrolled at a non-high school campus who are receiving free lunch. A non-high school campus is any campus that does not exclusively serve grades 9–12.

Percent limited English proficiency, non-high school. The percent of students enrolled at a non-high school identified as limited English proficient by the Language Proficiency Assessment Committee (LPAC) according to criteria established in the Texas Administrative Code. A non-high school campus is any campus that does not exclusively serve grades 9–12.

Percent special education (less severe). the percent of students identified as special education students and described as having learning or speech disabilities, but no additional disabilities. These data were provided by TEA.

Percent special education (more severe). The percent of students identified as special education students and having disabilities other than learning or speech disabilities. These data were provided by TEA.

Percent high school. The sum of district enrollment in grades 9–12 divided by total district enrollment. These data come from AEIS.

Miles to nearest major metro center. Miles to the nearest major metro area for each district was calculated as the pupil-weighted average distance from each campus to the geographic center of the nearest county with a population of at least 650,000 people. (According to the 2000 Census, there are six major metropolitan areas in Texas fitting this description—Austin, Dallas, El Paso, Fort Worth, Houston and San Antonio.) Distances are calculated using latitude and longitude information. The latitude and longitude of county centers come from the U.S. Census. Where available, latitude and longitude information for campuses are taken from the National Center For Education Statistics' Common Core Database (CCD). The remaining campuses are assigned latitudes and longitudes according to the zip codes at their street address. Using zip codes to identify geographic location can be problematic if the zip code area is large, but is unavoidable given that the CCD fails to identify the location of roughly one-third of Texas campuses. Comparing the latitude and longitude assigned by CCD to those implied by zip codes indicates that the average difference between the two locational indicators is less than three miles, but that it can reach 25 miles in some parts of the state. We censored the data at a lower bound of 10 miles due to concern that the geographic center of a county does not necessarily reflect the population center and our belief that districts in the range from zero to10 miles from the geographic center of the populous county were equally urban.

Predicted monthly salary for beginning teachers. While actual salary averages are an appropriate measure of price for estimation of a cost function, they are not appropriate for construction of a cost index. Only uncontrollable variations in compensation should be used to measure the cost of providing educational services. Therefore, when we calculate the cost function index or evaluate marginal effects, we use predicted teacher wages. For each district, the predicted compensation of the typical Texas teacher comes from the Teacher Fixed Effects Salary and Benefits model described in the companion report "Adjusting for Geographic Variations in Teacher Compensation: Technical Supplement." As the name indicates, the model uses an individual fixed effects methodology to separate variations in compensation that arise from uncontrollable local characteristics (such as the average price of single-family housing) from variations in compensation that arise from school district choices about the characteristics of their personnel. To adjust for differences between the salary and benefits of the typical Texas teacher and salary of beginning teachers, we multiply the predicted teacher compensation by 0.72292734 (.7739345), the ratio of average beginning salaries to average predicted compensation in 2002 (2004).

Predicted monthly salary for auxiliary workers. As with teacher wages, it is not appropriate to use actual salaries for teacher aides and auxiliary workers when calculating the cost function index or evaluating marginal effects. Therefore we use the predicted compensation of workers who are not teachers, administrators or support staff. For each district, the predicted wage comes from the Auxiliary Fixed Effects model described in the companion report "Adjusting for Geographic Variations in Teacher Compensation: Technical Supplement."

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